

Constant-Time Distributed Scheduling Policies for Ad Hoc Wireless Networks

Xiaojun Lin and Shahzada B. Rasool

School of Electrical and Computer Engineering, Purdue University
West Lafayette, IN 47907, U.S.A.
{linx, srasool}@ecn.purdue.edu.

Abstract— We propose two new distributed scheduling policies for ad hoc wireless networks that can achieve provable capacity regions. Known scheduling policies that guarantee comparable capacity regions are either centralized or need computation time that increases with the size of the network. In contrast, the unique feature of the proposed distributed scheduling policies is that they are constant-time policies, i.e., the time needed for computing a schedule is independent of the network size. Hence, they can be easily deployed in large networks.

I. INTRODUCTION

In this paper, we study the link scheduling problem in ad hoc wireless networks. In wireless networks, the radio transmissions at different links can interfere with each other. Hence, in order to achieve the optimal capacity, it is usually more efficient to only use a subset of the radio links at each time [1]. Determining which subset of links should be active at each time becomes the link scheduling problem, which is mainly at the MAC layer in the OSI reference model.

Good scheduling policies are those that can achieve large capacity regions and can be easily computed. Consider a wireless network with L links, and let λ_l be the data rate offered to link l . Let $\vec{\lambda} = [\lambda_1, \dots, \lambda_L]$. The *capacity region* under a particular scheduling policy is the set of data-rate vectors $\vec{\lambda}$ that the scheduling policy can support while keeping the queues at all links finite. A scheduling policy is said to be *throughput-optimal* if it can achieve the largest possible capacity region. Known throughput-optimal policies require solving a global optimization problem at each time [2], [3], [4], [5]. Such scheduling policies are inappropriate for ad hoc networks because the distributive nature of these networks requires simple and decentralized scheduling solutions. Recently, a number of distributed scheduling policies have been proposed in the literature [6], [7], [8], [9], [10]. Since the capacity region under a distributed policy is typically smaller than the optimal one achieved by the throughput-optimal policy, we define the *efficiency ratio* of such a sub-optimal scheduling policy as the largest number γ such that, given any network topology, for any $\vec{\lambda}$ that can be supported by the throughput-optimal policy, this policy can support $\gamma\vec{\lambda}$. In other words, the scheduling policy with efficiency ratio γ can achieve at least γ fraction of the optimal capacity region. Related works have studied a number of distributed scheduling policies that are

shown to achieve provable efficiency ratios. For example, the maximal matching policy has been shown to achieve an efficiency ratio of no less than $1/2$ under the node-exclusive interference mode [6]. Similar maximal scheduling policies are also studied under the bidirectional-equal-power model and the two-hop interference model [7], [8], [9], [10], where different bounds on the efficiency ratio are derived.

The problem with these existing distributed scheduling policies, however, is that the time needed to compute a schedule still increases with the size of the network. For example, one of the best known distributed algorithms in graph theory can compute maximal matching on a graph in $O(\log^4 N)$ rounds, where N is the total number of vertices in the graph [11]. Note that we need to be extra careful in interpreting this type of results under the context of ad hoc networks, because these distributed algorithms assume reliable message passing among neighboring nodes in each round of the computation. In ad hoc networks, these messages themselves can interfere with each other. Hence, it is likely that the actual amount of time needed for computing maximal matching in ad hoc networks can be much larger than $O(\log^4 N)$.

In this paper, we propose two new distributed scheduling policies. A unique feature of these new policies is that they are *constant-time* policies, i.e., the time needed to compute a schedule is independent of the size of the network. In fact, our proposed policies only require *one* round of computation. Hence, they are more scalable and easier to implement in large networks. We will provide constant-time distributed scheduling policies for two types of interference models, i.e., the node-exclusive interference model and the two-hop interference model. We will show that our proposed scheduling policies can achieve comparable efficiency ratios as some of the non-constant-time policies in the literature.

We believe that the results in the paper offer new insights for the design of simple and efficient scheduling policies for ad hoc networks. The rest of the paper is organized as follows. In Section II, we outline the network model and review related results. In Sections III and IV, we will propose the constant-time distributed scheduling policies for the node-exclusive interference model and the two-hop interference model, respectively, and derive their efficiency ratios. We discuss implementation issues in Section V, and present simulation results in Section VI. Then we conclude.

II. THE SYSTEM MODEL

Consider a wireless network with N nodes and L links. Each link corresponds to a pair of transmitter node and receiver node. Let $b(l)$ and $e(l)$ denote the transmitter node and the receiver node, respectively, of link l . Two nodes are one-hop neighbors of each other if they are the end-points of a common link. For each node i , let $E(l)$ denote the set of links that connect to the one-hop neighbors of node i , i.e., $E(l)$ is the set of links that node i either acts as a transmitter or as a receiver. Two links are *one-hop* neighbors of each other if they share a common node. Two links are *two-hop* neighbors of each other if they have a common one-hop neighboring link. For each link l , let $N^1(l)$ denote the set of one-hop neighbors of link l , i.e., $N^1(l) = [E(b(l)) \cup E(e(l))] \setminus \{l\}$. Further, let $N^2(l)$ denote the set of two-hop neighbors, i.e., $N^2(l) = \bigcup_{k \in N^1(l)} N^1(k) \setminus \{l\}$.

We first assume a single-hop traffic model, i.e., each packet only needs to traverse one of the L links and then leave the system. (The extension to the multi-hop case is treated in Section V.) We assume that time is divided into slots of unit length. Let $A_l(t)$ denote the number of packets arrive at link l at time slot t . We assume that packets are of unit length, and the packet arrival process $A_l(t)$ is stationary and ergodic.

We will study two types of interference models that govern the radio transmission. In both models, we say that two links interfere with each other if they cannot transmit data together. Under the *node-exclusive interference model*, each link l interferes with all of its one-hop neighboring links. Under the *two-hop interference model*, each link l interferes with all of its two-hop neighboring links. In both models, if the above interference constraints are satisfied, an active link l can transfer c_l packets within the time slot. We further assume that the system has carrier-sensing capabilities. In particular, under the one-hop interference model, we assume that all the one-hop neighboring links of link l can sense the transmission at link l . Under the two-hop interference model, we assume that all one-hop neighboring nodes of node i can sense the transmission from node i .

Remark: The node-exclusive interference model can be viewed as a generalization of the bipartite graph model for modeling high-speed packet switches [12], [13]. It has been used in [6], [14], [15] to model wireless networks. While this is a somewhat simplified model, the main results can often be readily generalized to other more complex interference models, e.g., the two-hop interference model. Note also that the latter model is very close to the interference model that IEEE 802.11 DCF (Distributed Coordination Function) deals with [10].

At time slot t , let $\mathcal{M}(t)$ denote the outcome of the scheduling policy, which is defined as the set of non-interfering links that are chosen to be active at time t . Let $D_l(t)$ denote the number of packets that link l can serve at time slot t . Then $D_l(t) = c_l$ if $l \in \mathcal{M}(t)$, and $D_l(t) = 0$ otherwise. Let $Q_l(t)$ denote the number of packets queued at link l at the beginning of time slot t , then the evolution of $Q_l(t)$ is

governed by

$$Q_l(t+1) = [Q_l(t) + A_l(t) - D_l(t)]^+, \quad (1)$$

where $[\cdot]^+$ denote the projection to $[0, +\infty)$.

We say that the system is *stable* if the queue lengths at all links remain finite [3], i.e.

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbf{1}_{\{\sum_{l=1}^L Q_l(t) > \eta\}} \rightarrow 0, \text{ almost surely as } \eta \rightarrow \infty. \quad (2)$$

Let λ_l be the mean packet arrival rate at link l , i.e., $\lambda_l = \mathbf{E}[A_l(t)]$. Let $\vec{\lambda} = [\lambda_1, \dots, \lambda_L]$. As we defined in the Introduction, the *capacity region* under a particular scheduling policy is the set of $\vec{\lambda}$ such that the system remains stable. The optimal capacity region Ω is the supremum of the capacity regions of all scheduling policies. A scheduling policy is *throughput-optimal* if it can achieve the optimal capacity region Ω . The *efficiency ratio* of a (possibly sub-optimal) scheduling policy is the largest number γ such that the scheduling policy can stabilize the system under any load $\vec{\lambda} \in \gamma\Omega$. By definition, a throughput-optimal scheduling policy has an efficiency ratio of 1.

A. Related Results

1) *Scheduling Policies for the Node-Exclusive Interference Model:* One of the known throughput-optimal scheduling policies under the node-exclusive interference model computes the set $\mathcal{M}(t)$ of non-interfering links at time-slot t such that $\mathcal{M}(t)$ maximizes the sum of the queue-weighted-rates

$$\sum_{l \in \mathcal{M}(t)} Q_l(t)c_l. \quad (3)$$

This scheduling policy is a direct application of the more general result in [2], [3], [4], [5]. The resulted schedule corresponds a **Maximum-Weighted-Matching (MWM)** of the underlying graph, where the weight of each link is $Q_l(t)c_l$. (Note that a *matching* is a subset of the links such that no two links share the same node. The *weight* of a matching is the total weight over all links belonging to the matching. A *maximum-weighted-matching (MWM)* is the matching with the maximum weight.) An $O(N^3)$ -complexity centralized algorithm for MWM can be found in [17], where N is the number of nodes. On the other hand, the following much simpler algorithm can be used to compute a suboptimal schedule that corresponds to a **Greedy Maximal Matching (GMM)** [13]: Start from an empty schedule; From all possible links, pick the link with the largest weight $Q_l(t)c_l$; Add this link to the schedule; Remove all links that are incident with either the transmitter node or the receiver node of link l ; Pick the link with the largest weight $Q_l(t)c_l$ from the *remaining* links, and add to the schedule; Continue until there are no links left. The above centralized GMM algorithm has only $O(L \log L)$ -complexity (where L is the number of links), and is much easier to implement than MWM. Using the technique in Theorem 10 of [13], we can show that the GMM policy achieves an efficiency ratio no less than $1/2$. There also exist distributed

algorithms that can compute the GMM schedule in $O(L)$ rounds [18].

The optimal capacity region Ω under the node-exclusive model is known to be bounded by [16]:

$$\frac{2}{3}\Psi_0 \subseteq \Omega \subseteq \Psi_0, \quad (4)$$

where

$$\Psi_0 = \left\{ \vec{\lambda} \left| \sum_{l \in E(i)} \frac{\lambda_l}{c_l} \leq 1, \text{ for all nodes } i \right. \right\}. \quad (5)$$

The following **Maximal-Matching (MM)** policy can be shown to achieve a capacity region of $\Psi_0/2$, and thus also has an efficiency ratio of at least $1/2$ [6]. The MM policy simply picks a set $\mathcal{M}(t)$ of non-interfering links such that no more links can be added to $\mathcal{M}(t)$ without violating the node-exclusive interference constraint. To be precise, a Maximal Matching $\mathcal{M}(t)$ is a set of non-interfering links such that: (a) $Q_l(t) \geq 1$ for all $l \in \mathcal{M}$, and (b) for each link l in the network, either $Q_l(t) < 1$ or some links in $E(b(l)) \cup E(e(l))$ is included in \mathcal{M} . The distributed algorithm in [11] can compute a maximal matching in $O(\log^4 N)$ rounds.

2) *Scheduling Policies for the Two-Hop Interference Model*: Under the top-hop interference model, the optimal capacity region Ω is bounded by $\Omega \subseteq \Psi'_0$, where

$$\Psi'_0 = \left\{ \vec{\lambda} \left| \sum_{k \in N^1(l)} \frac{\lambda_l}{c_l} \leq 1, \text{ for all } l \right. \right\}. \quad (6)$$

The policy that maximizes (3) among all set $\mathcal{M}(t)$ of non-interfering links is still throughput-optimal. However, finding such a set $\mathcal{M}(t)$ is generally an NP-Complete problem [19], [20]. Greedy Maximal Scheduling policy and Maximal Scheduling policy under the two-hop interference model can be defined analogously to the GMM and MM policies, respectively, under the node-exclusive interference model. The efficiency ratio of these policies is $1/\widehat{N}^1$, where $\widehat{N}^1 \triangleq 1/\max_l |N^1(l)|$ is the maximum number of one-hop neighboring links of any link [7], [8], [9], [10]. Neither of the two policies are constant-time scheduling policies.

III. A CONSTANT-TIME DISTRIBUTED SCHEDULING POLICY FOR THE NODE-EXCLUSIVE INTERFERENCE MODEL

None of the distributed scheduling policies in Section II-A can compute a schedule in constant time (i.e., in a time that is independent of the network size). In this section, we propose a new distributed scheduling policy for the node-exclusive interference model that only needs $O(1)$ time to compute a schedule, and we will show that it achieves an efficiency ratio at least close to $1/3$. The new policy operates as follows:

Constant-Time Distributed Scheduling Policy P :

At each time slot t :

- Each link l computes a probability $p_l(t)$ based on its own queue-length and that of its one-hop neighboring

links as follows:

$$p_l(t) = \frac{Q_l(t)}{\max\left[\sum_{k \in E(b(l))} \frac{Q_k(t)}{c_k}, \sum_{k \in E(e(l))} \frac{Q_k(t)}{c_k}\right]}. \quad (7)$$

- Each link l attempts transmission with probability $p_l(t)$, and does not attempt transmission with probability $1 - p_l(t)$. For those links that attempt transmission, each of them randomly and independently chooses a backoff time uniformly from $\{0, 1, \dots, M - 1\}$, where M is a system-wide positive integer constant. We assume that all backoff timers start at the beginning of the time slot. When a link's backoff timer expires, the transmission at the link starts, provided that it has not overheard (i.e., through carrier-sensing) any other transmission from its one-hop neighboring links. Hence, the link l whose backoff timer expires ahead of all of its interfering links will win, and will be able to successfully transfer packets in the time-slot. It is possible that two or more links' backoff timers expire at the same time, in which case collision occurs and none of the interfering links can transfer packets in time-slot t .

We make the following remarks before we derive the efficiency ratio of Policy P .

Random Backoff: Note that the random backoff procedure in the second step of the policy is typical in random access protocols (e.g., IEEE 802.11 and Ethernet) to avoid excessive contention. In practical implementations, the actual backoff time depends both on the constant M and on how long each unit of backoff time lasts. In practice, due to propagation and processing delays, the length of each unit of backoff time cannot be arbitrarily small. For example, in IEEE 802.11, each unit of backoff time lasts $20\mu s$. Therefore, in order to compute the schedule in constant time, we need to provide an upper bound on M . In Section III-A, we will see how the efficiency ratio of Policy P depends on M .

Attempt Probability: The choice of the attempt probability p_l is also essential to obtain constant-time scheduling policies with an efficiency ratio independent of the network topology. Otherwise, if p_l is lower bounded by a constant, we can show below that the throughput of the system may drop to zero. To see this, consider the simple example of L nodes transmitting to a common receiver. Hence, the L links interfere with each other. Given a fixed value of M , the probability that any one of the L links can successfully transfer data in a given time slot is bounded from above by the sum of the probability that one link wins with backoff time equal to 1, plus the probability that no links have backoff times equal to 1, i.e., bounded by

$$\sum_{l=1}^L \frac{p_l}{M} \prod_{k \neq l} \left(1 - \frac{p_k}{M}\right) + \prod_{l=1}^L \left(1 - \frac{p_l}{M}\right).$$

If p_l is bounded from below by a constant p , then the above bound will go to zero as $L \rightarrow \infty$. Hence, the total system throughput will drop to zero for any fixed value of M . On the other hand, since in (7) we set the attempt probability

inversely proportional to the sum of the queue-length at the interfering links, we reduce the chance of contention in the neighborhood. As we will see in Section III-A, a fixed value of M will then be sufficient to guarantee a fixed efficiency ratio for *arbitrary* network topologies.

Finally, we note that Policy P can be viewed as an extension of the Longest-Queue-Driven (LQD) scheduling algorithm from the switching literature [13]. However, there are two key differences: (a) in the switching literature, the network topology is a bipartite graph, while ad hoc network topology is non-bipartite; (b) in the switching literature, the transmitting nodes (i.e., input ports) and receiving node (i.e., output ports) are determined a priori, while in ad hoc networks a node can alternate its role as transmitter or receiver from time-slot to time-slot. The proposed policy P has carefully accounted for these differences through the random backoff phase in the second part of the policy.

A. The Efficiency Ratio of Policy P

We next show that the efficiency ratio of the above policy P is at least close to $1/3$. We start with some definitions.

Definition 1: Let \vec{x} be a component-wise positive vector in \mathbb{R}^L and let Θ be a convex, closed and bounded subset in the positive quadrant of \mathbb{R}^L . Assume that Θ contains a neighborhood of the origin. The norm of \vec{x} with respect to Θ is given by

$$\|\vec{x}\|_{\Theta} = \frac{1}{\max\{k | k \geq 0, k\vec{x} \in \Theta\}}.$$

The normalized vector $\tilde{\mathbf{x}}$ of \vec{x} with respect to Θ is defined as

$$\tilde{\mathbf{x}} = \frac{\vec{x}}{\|\vec{x}\|_{\Theta}}, \quad (8)$$

i.e., $\tilde{\mathbf{x}}$ is the longest vector in Θ that is in the same direction as \vec{x} .

The following lemma shows that the above-defined norm is indeed a *norm*. Denote $\vec{y} \succeq \vec{x}$ if \vec{y} is component-wise greater than or equal to \vec{x} .

- Lemma 1:*
- 1) $\|\vec{x}\| \geq 0$.
 - 2) $\|\tilde{\mathbf{x}}\| = 1$.
 - 3) If $\vec{y} = \alpha\vec{x}$, where α is a real number, then $\|\vec{y}\| = |\alpha| \|\vec{x}\|$.
 - 4) If $\vec{y} \succeq \vec{x}$, then $\|\vec{y}\| \geq \|\vec{x}\|$.
 - 5) $\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$.

Proof: Parts 1 to 4 follow trivially from the definition. To show Part 5, let $k_1 = \|\vec{x}\|$ and $k_2 = \|\vec{y}\|$. Thus,

$$\frac{\vec{x}}{k_1} \in \Theta, \frac{\vec{y}}{k_2} \in \Theta.$$

Hence, by the convexity of Θ , we have,

$$\frac{\vec{x} + \vec{y}}{k_1 + k_2} = \frac{\vec{x}}{k_1} \frac{k_1}{k_1 + k_2} + \frac{\vec{y}}{k_2} \frac{k_2}{k_1 + k_2} \in \Theta.$$

By the definition of the $\|\vec{x} + \vec{y}\|$, we thus have

$$\|\vec{x} + \vec{y}\| \leq k_1 + k_2. \quad \blacksquare$$

Now, let $\vec{Q}(t) = [Q_1(t), \dots, Q_L(t)]$, where $Q_l(t)$ denote the queue length of link l at the beginning of time slot t . Let $d_l^0(t) = p_l(t)c_l$, and let $\vec{d}^0 = [d_1^0, \dots, d_L^0]$.

Lemma 2: $\vec{d}^0(t) \succeq \tilde{\mathbf{Q}}(t)$, where the normalization is with respect to Ψ_0 .

Proof: From (5) and (8), we have $\tilde{\mathbf{Q}}(t) = k_0 \vec{Q}(t)$, where k_0 is the largest positive number k that satisfies,

$$\sum_{l \in E(i)} \frac{kQ_l(t)}{c_l} \leq 1 \text{ for all } i.$$

Hence,

$$k_0 = \frac{1}{\max_i \sum_{l \in E(i)} \frac{Q_l(t)}{c_l}}.$$

Using (7), we have $p_l(t) \geq \frac{k_0 Q_l(t)}{c_l}$. The result then follows. \blacksquare

Lemma 2 shows that, if links that attempt transmission were to win every time, then the expected amount of service provided by link l at time-slot t is component-wise no less than $\tilde{\mathbf{Q}}(t)$. However, due to the random-backoff procedure in the second part of Policy P , only a few links that attempt transmission will win. We next show that, if a link attempts transmission, the conditional probability that it wins is no less than $\frac{1}{3} - \frac{1}{M}$. In fact, we will prove a more general result as follows. Fix a particular link 0. Label its interfering links as $1, 2, \dots, K$. Let x_k denote the the probability that the k -th interfering link attempts transmission. Assume that all links follow the random backoff procedure in the second part of Policy P .

Lemma 3: If $\sum_{k=1}^K x_k \leq H$, where $H \geq 0$, then the conditional probability that link 0 wins, conditioned on it attempts transmission, is no less than $\frac{1}{H+1} - \frac{1}{M}$.

Proof: Condition the following derivation on the event that link 0 attempts transmission. Let Y be the random variable that denote the backoff time of link 0, Conditioned on $Y = y$, the probability that link 0 wins is no less than the probability that all K interfering links either do not attempting transmission, or have backoff time greater than y . Note that each interfering link attempts transmission and chooses its backoff time independently. Let S denote the event that link 0 wins. We thus have,

$$\begin{aligned} \mathbf{P}[S|Y = y] &\geq \prod_{k=1}^K \left[\frac{(M-1-y)x_k}{M} + (1-x_k) \right] \\ &= \prod_{k=1}^K \left[1 - \frac{y+1}{M} x_k \right]. \end{aligned}$$

Since Y is uniformly distributed among $\{0, \dots, M-1\}$, we have

$$\begin{aligned} \mathbf{P}[S] &= \sum_{y=0}^{M-1} \frac{\mathbf{P}[S|Y = y]}{M} \\ &\geq \sum_{y=0}^{M-1} \frac{1}{M} \prod_{k=1}^K \left[1 - \frac{y+1}{M} x_k \right]. \end{aligned}$$

Since $\prod_{k=1}^K (1 - ux_k)$ is decreasing in u , we have,

$$\begin{aligned} \mathbf{P}[S] &\geq \int_{\frac{1}{M}}^1 \prod_{k=1}^K (1 - ux_k) du \\ &\geq \int_0^1 \prod_{k=1}^K (1 - ux_k) du - \frac{1}{M}. \end{aligned} \quad (9)$$

By comparing the derivatives, we can show that

$$\prod_{k=1}^K (1 - ux_k) \geq (1 - u)^H.$$

Hence,

$$\mathbf{P}[S] \geq \int_0^1 (1 - u)^H du - \frac{1}{M} = \frac{1}{H+1} - \frac{1}{M}.$$

Remark: A special case of Lemma 3 that corresponds to $H = 1$ and $M = \infty$ was shown in Theorem 5 of [13]. Here we provide a more general result using a much different proof technique.

Under Policy P , we infer from (7) that, for any link l , the attempt probabilities of its one-hop neighboring links must satisfy $\sum_{k \in E(b(l))} p_k(t) \leq 1$ and $\sum_{k \in E(e(l))} p_k(t) \leq 1$. Hence, the sum of the attempt probabilities over its interfering links is no greater than 2. We thus obtain the following corollary to Lemma 3.

Corollary 4: Under Policy P , the conditional probability that link l wins, conditioned on it attempts transmission, is no less than $\frac{1}{3} - \frac{1}{M}$.

Using Lemma 2 and Corollary 4, we thus conclude that the average service provided by link l at time-slot t is no less than $\tilde{\mathbf{Q}}(t)(\frac{1}{3} - \frac{1}{M})$. We can now prove our main result.

Proposition 5: Under Policy P , the network is stable when $\vec{\lambda}$ lies strictly inside the set $(\frac{1}{3} - \frac{1}{M})\Psi_0$.

Proof: We will prove stability by finding a Lyapunov function with negative drift for the fluid model of the system. The fluid model is defined as follows [12], [21]. We first interpolate the values of $Q_l(t)$ to all non-negative real number t by linear interpolation between $[t]$ and $[t] + 1$ (where $[t]$ denote the largest integer no greater than t). We also define the values of $A_l(t)$ and $D_l(t)$ for all real number t by letting $A_l(t) = A_l([t])$, and $D_l(t) = D_l([t])$. Then, using the techniques of Theorem 4.1 of [21], we can show that, for almost all sample paths and for any positive sequence $x_n \rightarrow \infty$, there exists a subsequence x_{n_j} with $x_{n_j} \rightarrow \infty$ such that

$$\begin{aligned} \frac{1}{x_{n_j}} Q_l(x_{n_j} t) &\rightarrow q_l(t) \text{ for all } l, \\ \frac{1}{x_{n_j}} \int_0^{x_{n_j} t} A_l(s) ds &\rightarrow \lambda_l t \text{ for all } l, \\ \frac{1}{x_{n_j}} \int_0^{x_{n_j} t} D_l(s) ds &\rightarrow \int_0^t d_l(s) ds \text{ for all } l, \end{aligned} \quad (10)$$

uniformly over compact intervals. Further, let $\vec{q}(t) = [q_1(t), q_2(t), \dots, q_L(t)]$ and $\vec{d}(t) = [d_1(t), d_2(t), \dots, d_L(t)]$.

Using Lemmas 2 and 3, and the techniques of Theorem 4.1 of [21] again, we can show that the limits $\vec{q}(t)$ and $\vec{d}(t)$ satisfy the following set of equations: for all l ,

$$\begin{aligned} \frac{d}{dt} q_l(t) &= \begin{cases} \lambda_l - d_l(t), & \text{if } \lambda_l - d_l(t) \geq 0, \\ & \text{or } q_l(t) > 0 \\ 0, & \text{otherwise} \end{cases} \\ q_l(t) &\geq 0, \text{ and } \vec{d}(t) \succeq \tilde{\mathbf{q}}(t) \left(\frac{1}{3} - \frac{1}{M} \right), \end{aligned} \quad (11)$$

where $\tilde{\mathbf{q}}(t)$ is the normalized vector of $\vec{q}(t)$ with respect to Ψ_0 . Any such limit $[\vec{q}(t), \vec{d}(t)]$ is called a *fluid limit* of the system. We say that a fluid limit model of the system is stable if there exists a constant T that depends only on the network topology, the arrival rates λ_l and the active link capacities c_l , such that for *any* fluid limit with $\|\vec{q}(0)\| = 1$, we have $\|\vec{q}(t)\| = 0$ for all $t \geq T$ [12], [21].

We now use the following Lyapunov function

$$V(\vec{q}(t)) = \|\vec{q}(t)\|$$

to show that the fluid limit model of the system is stable, where the norm is defined with respect to Ψ_0 . For any small positive number δt , we have,

$$\begin{aligned} &V(\vec{q}(t + \delta t)) \\ &\leq \|\vec{q}(t) + \vec{\lambda} \delta t - \vec{d}(t) \delta t\| + o(\delta t) \\ &\leq \|\vec{q}(t) - \vec{d}(t) \delta t\| + \|\vec{\lambda}\| \delta t + o(\delta t) \\ &\quad \text{(by Part 5 of Lemma 1)} \\ &\leq \|\vec{q}(t) - \tilde{\mathbf{q}}(t) \left(\frac{1}{3} - \frac{1}{M} \right) \delta t\| + \|\vec{\lambda}\| \delta t + o(\delta t) \\ &\quad \text{(by (11) and Part 4 of Lemma 1)} \\ &= \|\vec{q}(t)\| - \left(\frac{1}{3} - \frac{1}{M} \right) \delta t + \|\vec{\lambda}\| \delta t + o(\delta t) \\ &\quad \text{(by Parts 2 and 3 of Lemma 1)}. \end{aligned}$$

When $\vec{\lambda}$ lies strictly inside the set $(\frac{1}{3} - \frac{1}{M})\Psi_0$, we have

$$\|\vec{\lambda}\| \leq \left(\frac{1}{3} - \frac{1}{M} \right) - \beta,$$

for some constant $\beta > 0$. Hence, we have,

$$\frac{dV(\vec{q}(t))}{dt} \leq -\beta.$$

This then shows that the fluid limit model of the system is stable. By Theorem 4.2 of [21], the original system is also stable (i.e., positive Harris recurrent). ■

Remark: For any given $\epsilon > 0$, we can choose the maximum backoff time $M = 1/\epsilon$, which then ensures that the efficiency ratio of Policy P is no less than $1/3 - \epsilon$. Note that for each ϵ , the value of M is independent of the network topology. Hence, we have shown that Policy P only takes constant time and can guarantee an efficiency ratio close to $1/3$ for arbitrary network topologies. As $M \rightarrow \infty$, the guaranteed efficiency ratio goes to $1/3$. Hence, the difference ϵ is the loss in efficiency due to the constant-time requirement. We also note that the same technique for proving Proposition 5 can be used to establish the result of Theorems 6 and 7 in [13]. Compared with the proofs there, our construction of the Lyapunov function is new and much

easier to understand. Alternatively, the result of Proposition 5 may be shown using the Lyapunov function in [22].

IV. A CONSTANT-TIME DISTRIBUTED SCHEDULING POLICY FOR THE TWO-HOP INTERFERENCE MODEL

We next extend the constant-time distributed policy in the previous section to the two-hop interference model. Under the two-hop interference model, the best known distributed scheduling policy can guarantee an efficiency ratio of $1/\widehat{N}^1$, where $\widehat{N}^1 \triangleq \max_l |N^1(l)|$ is the maximum number of one-hop neighboring links for any link [7], [8], [9], [10]. The distributed policies in these prior works are not constant-time policies. We now propose a constant-time distributed scheduling policy Q that can guarantee a comparable efficiency ratio.

Constant-Time Distributed Scheduling Policy Q :

Let W be a positive number between 1 and \widehat{N}^1 . At each time slot t :

- Each link l computes a probability $p_l(t)$ based on its own queue-length and that of the interfering links, i.e.

$$p_l(t) = \frac{\frac{Q_l(t)}{c_l}}{\max_{k \in N^1(l)} \sum_{h \in N^1(k)} \frac{Q_h(t)}{c_h}} \times \min \left(1, \frac{W}{\max_{k \in N^1(l)} |N^1(k)|} \right).$$

- Each link l attempts transmission with probability $p_l(t)$, and does not attempt transmission with probability $1 - p_l(t)$. For those links that attempt transmission, each of them randomly chooses a backoff time uniformly from $\{0, 1, \dots, M - 1\}$. We assume that all backoff timers start at the beginning of the time slot. When the backoff time of a link l expires, the transmitter node $b(l)$ of link l will broadcast an RTS to all of its one-hop neighboring nodes, provided that node $b(l)$ has not overheard any RTS from these one-hop neighboring nodes. Once the receiver node $e(l)$ correctly receive the RTS, it will then respond with a CTS broadcasted to all of its neighboring nodes. Through this RTS-CTS procedure, the link l that sends out an RTS before any of its two-hop neighboring links will win. This link l can then transfer packets at the rate of c_l during the rest of the time slot. It is possible that two or more links in a two-hop neighborhood send out RTS together, in which case collision occurs and none of the interfering links can transfer data in time-slot t .

We can use similar techniques as in Section III to show that policy Q guarantees an efficiency ratio close to $1/\widehat{N}^1$. To see this, note that under the two-hop interference model, the optimal capacity region Ω is upper bounded by Ψ'_0 in (6). Define $d_l^0(t) = p_l(t)c_l$, and let $\vec{d}^0 = [d_1^0, \dots, d_L^0]$. Using the technique of Lemma 2, we have

$$\vec{d}^0(t) \succeq \frac{W}{\widehat{N}^1} \vec{Q}(t),$$

where $\vec{Q}(t)$ is the normalized vector of $\vec{Q}(t)$ with respect to Ψ'_0 . Further, for each link l , the sum of the attempt probabilities of its interfering links (i.e., its two-hop neighboring links) satisfies

$$\begin{aligned} \sum_{k \in N^2(l)} p_k &\leq \sum_{k \in N^1(l)} \sum_{h \in N^1(k)} p_h \\ &\leq \sum_{k \in N^1(l)} \frac{W}{N^1(l)} = W. \end{aligned}$$

Therefore, using Lemma 3 with $H = W$, and using the technique of Proposition 5, we can show the following main result.

Proposition 6: Under Policy Q , the network is stable when $\vec{\lambda}$ lies strictly inside the set $\frac{W}{\widehat{N}^1} \left(\frac{1}{1+W} - \frac{1}{M} \right) \Psi'_0$.

Remark: For any fixed W , by choosing $M \geq \frac{1}{(1+W)(2+W)}$, the efficiency ratio of Policy Q is at least $\frac{W}{(2+W)\widehat{N}^1}$. For each W , the value of M is independent of the network topology. Hence, we have shown that Policy Q only takes constant time. As $W \rightarrow \widehat{N}^1$ and $M \rightarrow \infty$, the guaranteed efficiency ratio goes to $1/(1 + \widehat{N}^1)$. The difference $\left(\frac{1}{1+\widehat{N}^1} - \frac{W}{(2+W)\widehat{N}^1} \right)$ is the loss of capacity due to the constant-time requirement.

V. EXTENSION TO MULTI-HOP NETWORKS AND DISCUSSIONS

In this section, we extend the constant-time scheduling policies of the previous sections to multi-hop networks, and also address the practical implementation issue of communicating the queue lengths. We will focus on the node-exclusive interference model (and Policy P), while the same methodology can be applied to the two-hop interference model (and Policy Q) as well.

A. Constant-Time Scheduling Policy for Multihop Wireless Networks

In Section II, we have assumed a single-hop traffic model, i.e., each packet only needs to traverse one of the L links and then leaves the system. We next extend policy P to multi-hop networks with fixed routing. Assume that there are S end-users in the system. Each user injects packets at the rate of x_s packets per time-slot. Assume that each user has a fixed path through the network, and let $[H_s^l]$ denote the routing matrix, where $H_s^l = 1$ if the path of user s traverse link l , and $H_s^l = 0$ otherwise. Thus, the aggregate data rate on link l , denoted by λ_l , is given by $\lambda_l = \sum_{s=1}^S H_s^l x_s$. Redefine the capacity region of the network under a particular scheduling policy to be the set of $\vec{x} = [x_1, \dots, x_S]$ such that the system can remain stable. Then the optimal capacity region Ω_M is upper bounded by,

$$\Omega_M \subset \left\{ \vec{x} \mid \left[\sum_{s=1}^S H_s^l x_s \right] \in \Psi_0 \right\},$$

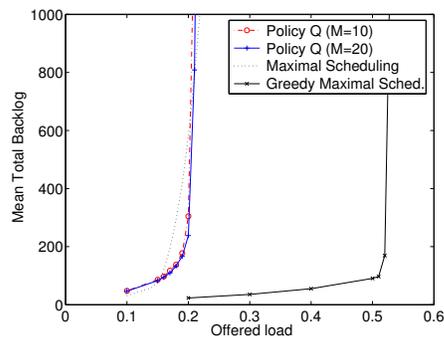


Fig. 3. Performance comparison under the two-hop interference model

shown that both scheduling policies can guarantee efficiency ratios comparable to other known distributed scheduling policies in the literature.

For future work, we plan to generalize the main techniques and results to other types of interference models, e.g., the bi-directional equal power model in [7]. We also note that Policy Q for the two-hop interference model operates in a very similar way as IEEE 802.11 DCF (Distributed Coordination Function). The main difference is: when there is excessive contention, IEEE 802.11 DCF will increase the backoff window exponentially; however, Policy Q will reduce the attempt probability, and keep the backoff window unchanged. It would be an interested direction for future work to explore the performance differences of these two approaches. Finally, in both Fig. 2 and Fig. 3 we observe that the performance of Greedy Maximal Scheduling policies are typically better than that of Policies P and Q . This may indicate that the efficiency ratio of Greedy Maximal Scheduling policies could have been under-estimated. Further, it would be interesting to study whether one can develop constant-time distributed scheduling policies that achieve comparable performance as these Greedy Maximal Scheduling policies.

REFERENCES

- [1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, March 2000.
- [2] L. Tassiulas and A. Ephremides, "Stability Properties of Constrained Queueing Systems and Scheduling Policies for Maximum Throughput in Multihop Radio Networks," *IEEE Transactions on Automatic Control*, vol. 37, no. 12, pp. 1936–1948, December 1992.
- [3] M. J. Neely, E. Modiano, and C. E. Rohrs, "Dynamic Power Allocation and Routing for Time Varying Wireless Networks," in *Proceedings of IEEE INFOCOM*, San Francisco, April 2003.
- [4] A. Eryilmaz, R. Srikant, and J. Perkins, "Stable Scheduling Policies for Fading Wireless Channels," *IEEE/ACM Transactions on Networking*, to appear.
- [5] R. L. Cruz and A. V. Santhanam, "Optimal Routing, Link Scheduling and Power Control in Multi-hop Wireless Networks," in *Proceedings of IEEE INFOCOM*, San Francisco, April 2003.
- [6] X. Lin and N. B. Shroff, "The Impact of Imperfect Scheduling on Cross-Layer Rate Control in Multihop Wireless Networks," in *Proceedings of IEEE INFOCOM*, Miami, FL, March 2005.
- [7] P. Chaporkar, K. Kar, and S. Sarkar, "Throughput Guarantees in Maximal Scheduling in Wireless Networks," in *Proceedings of 43rd Annual Allerton Conference on Communication, Control and Computing*, Monticello, IL, September 2005.
- [8] —, "Achieving Queue Length Stability Through Maximal Scheduling in Wireless Networks," in *Proceedings of Information Theory and Applications Inaugural Workshop*, University of California, San Diego, February 2006.
- [9] X. Wu, R. Srikant, and J. R. Perkins, "Queue-Length Stability of Maximal Greedy Schedules in Wireless Network," in *Proceedings of Information Theory and Applications Inaugural Workshop*, University of California, San Diego, February 2006.
- [10] X. Wu and R. Srikant, "Bounds on the Capacity Region of Multihop Wireless Networks Under Distributed Greedy Scheduling," in *Proceedings of IEEE INFOCOM*, Barcelona, Spain, April 2006.
- [11] M. Hanckowiak, M. Karonski, and A. Panconesi, "On the Distributed Complexity of Computing Maximal Matching," *SIAM Journal of Discrete Mathematics*, vol. 15, no. 1, pp. 41–57, 2001.
- [12] J. G. Dai and B. Prabhakar, "The Throughput of Data Switches with and without Speedup," in *Proceedings of IEEE INFOCOM*, Tel Aviv, Israel, March 2000, pp. 556–564.
- [13] E. Leonardi, M. Mellia, F. Neri, and M. A. Marsan, "On the Stability of Input-Queued Switches with Speed-Up," *IEEE/ACM Transactions on Networking*, vol. 9, no. 1, pp. 104–118, February 2001.
- [14] S. Sarkar and L. Tassiulas, "End-to-end Bandwidth Guarantees Through Fair Local Spectrum Share in Wireless Ad-hoc Networks," in *Proceedings of the IEEE Conference on Decision and Control*, Maui, Hawaii, December 2003.
- [15] Y. Yi and S. Shakkottai, "Hop-by-hop Congestion Control over a Wireless Multi-hop Network," in *Proceedings of IEEE INFOCOM*, Hong Kong, March 2004.
- [16] B. Hajek and G. Sasaki, "Link Scheduling in Polynomial Time," *IEEE Transactions on Information Theory*, vol. 34, no. 5, pp. 910–917, September 1988.
- [17] C. H. Papadimitriou and K. Steiglitz, *Combinatorial Optimization: Algorithms and Complexity*. Englewood Cliffs, New Jersey: Prentice-Hall, 1982.
- [18] D. E. Drake and S. Hougardy, "A Linear-Time Approximation Algorithm for Weighted Matchings in Graph," *ACM Transactions on Algorithms*, vol. 1, no. 1, pp. 107–122, July 2005.
- [19] G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Maximum Weighted Matching with Interference Constraints," in *IEEE International Workshop on Foundations and Algorithms For Wireless Networking*, Pisa, Italy, March 2006.
- [20] H. Balakrishnan, C. L. Barrett, V. S. A. Kumar, M. V. Marathe, and S. Thite, "The Distance-2 Matching Problem and its Relationship to the MAC-layer Capacity of Ad hoc Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 6, pp. 1069–1079, August 2004.
- [21] J. G. Dai, "On Positive Harris Recurrence of Multiclass Queueing Networks: A Unified Approach via Fluid Limit Models," *Annals of Applied Probability*, vol. 5, pp. 49–77, 1995.
- [22] A. Eryilmaz, R. Srikant, and J. R. Perkins, "Stable Scheduling Policies for Broadcast Channels," in *ISIT 2002*, Lausanne, Switzerland, June–July 2002.
- [23] S. H. Lu and P. R. Kumar, "Distributed Scheduling Based on Due Dates and Buffer Priorities," *IEEE Transactions on Automatic Control*, pp. 1406–1416, December 1991.
- [24] X. Lin and N. B. Shroff, "The Impact of Imperfect Scheduling on Cross-Layer Rate Control in Multihop Wireless Networks," *Technical Report, Purdue University*, <http://min.ecn.purdue.edu/~linx/papers.html>, 2004.