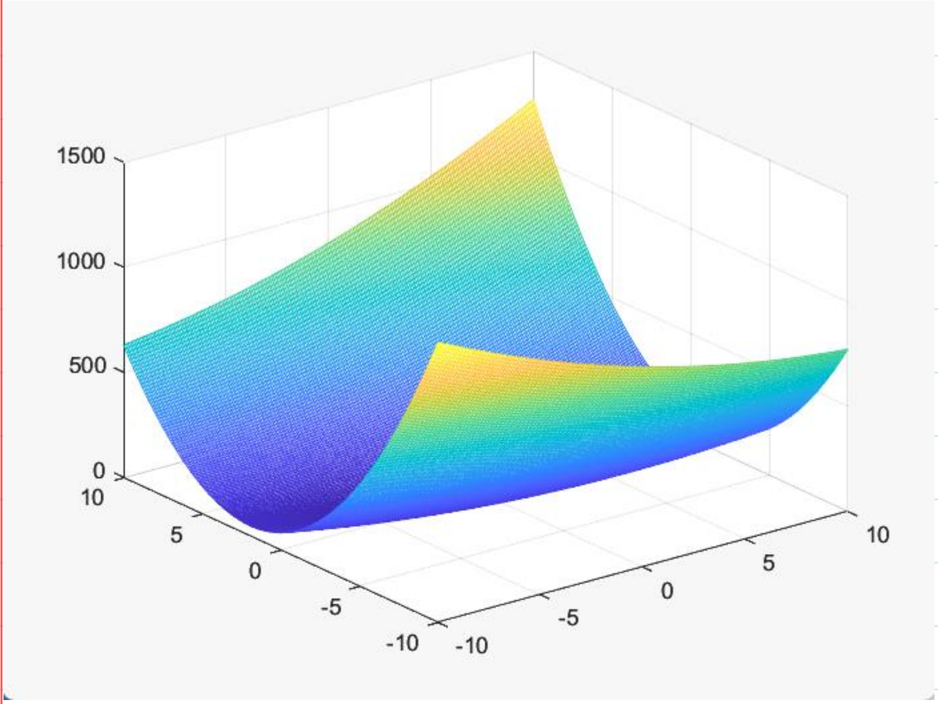
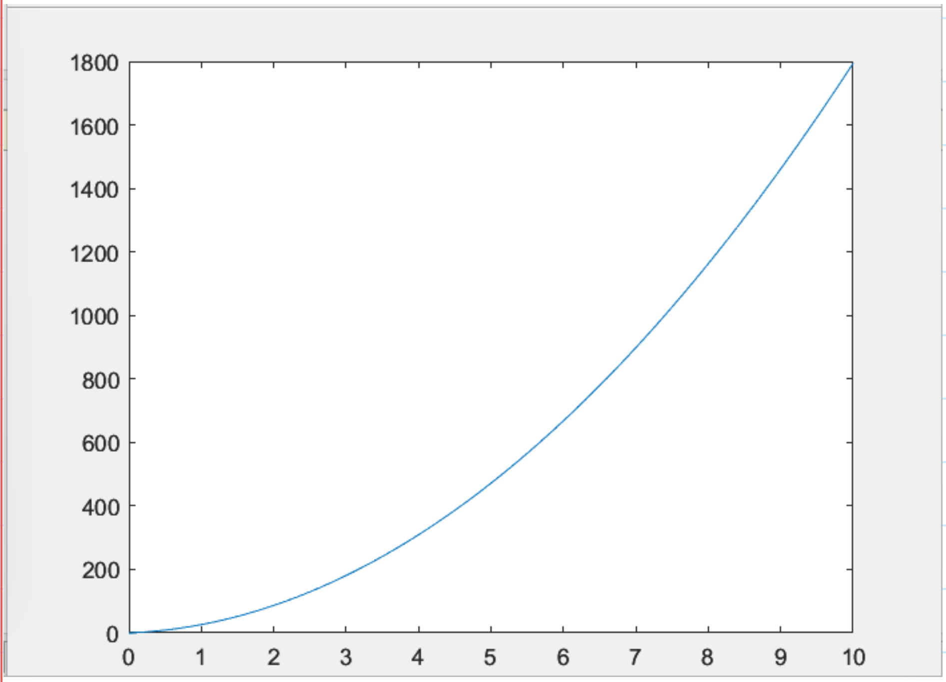
P1a

Thursday, March 30, 2023 1:35 PM

This is a 3D plot of the objective function, showing a convex function.



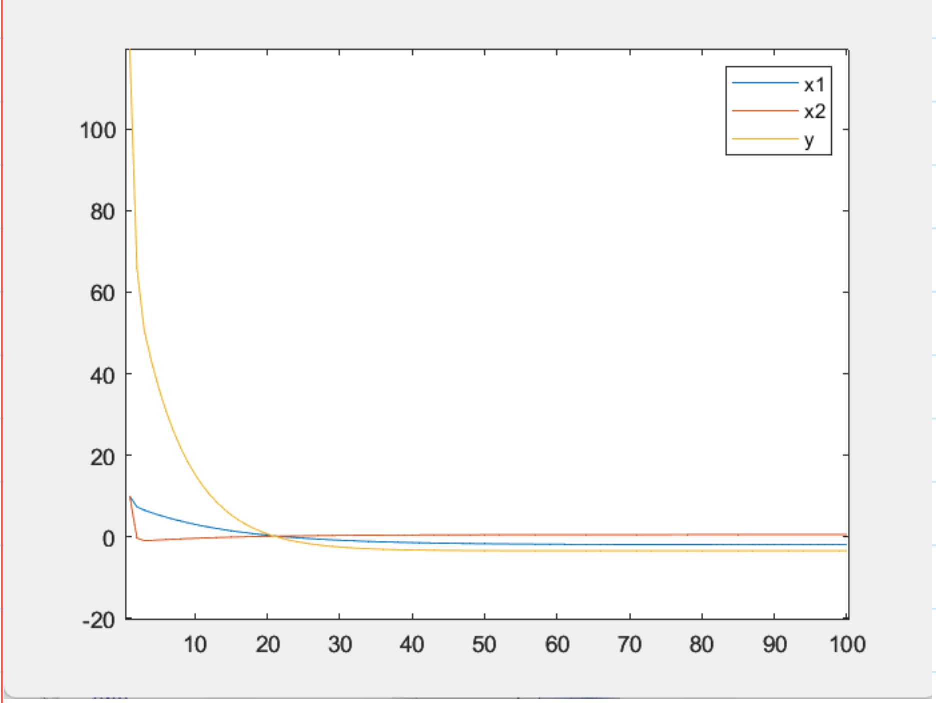
This is a restriction of the function on one random line, showing a convex function too.



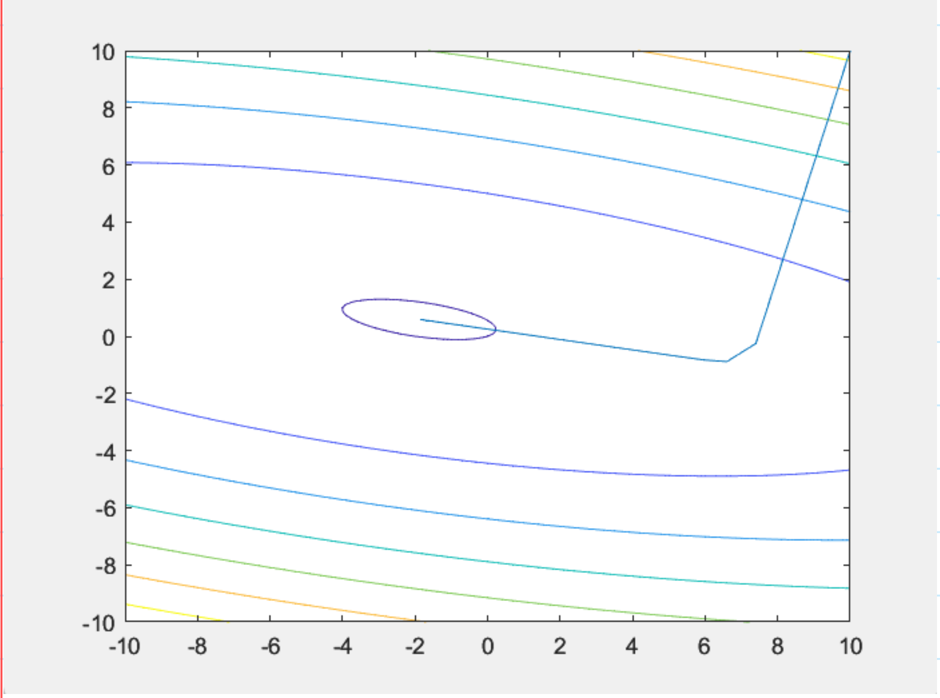
P1b

Thursday, March 30, 2023 1:38 PM

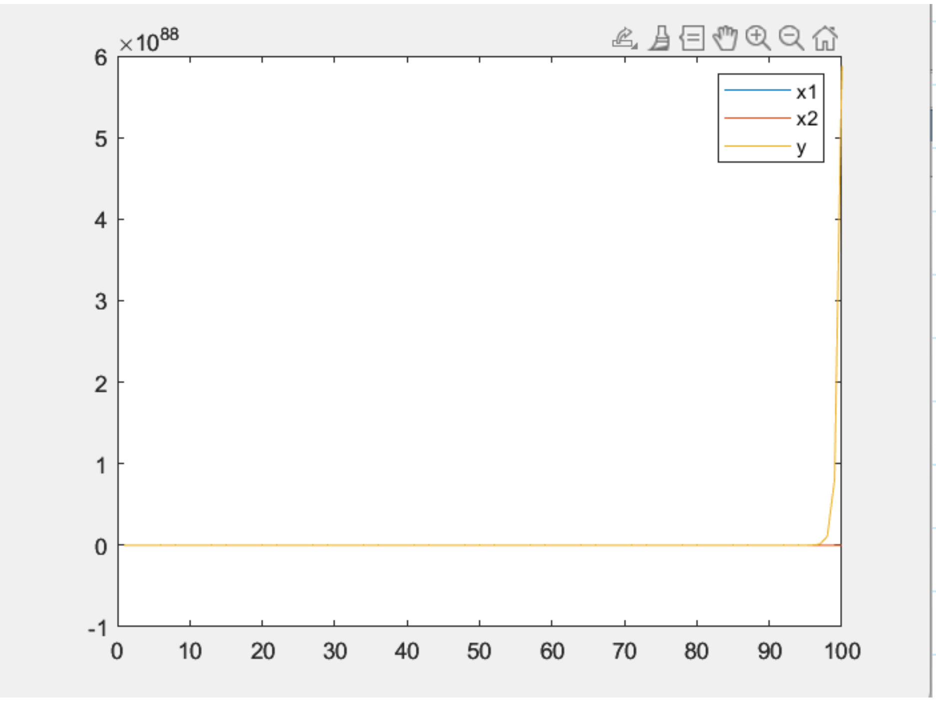
This shows the convergence of primal variables when stepsize=0.05. Convergence speed is about right.
The final point is at
x1=-1.888888, x2=0.592590



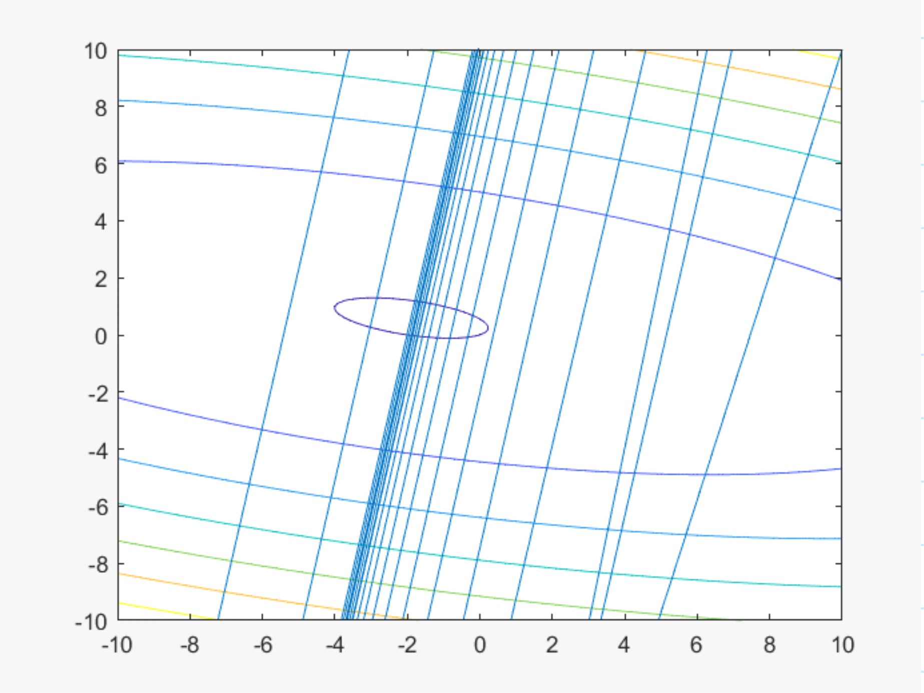
This is the contour plot:



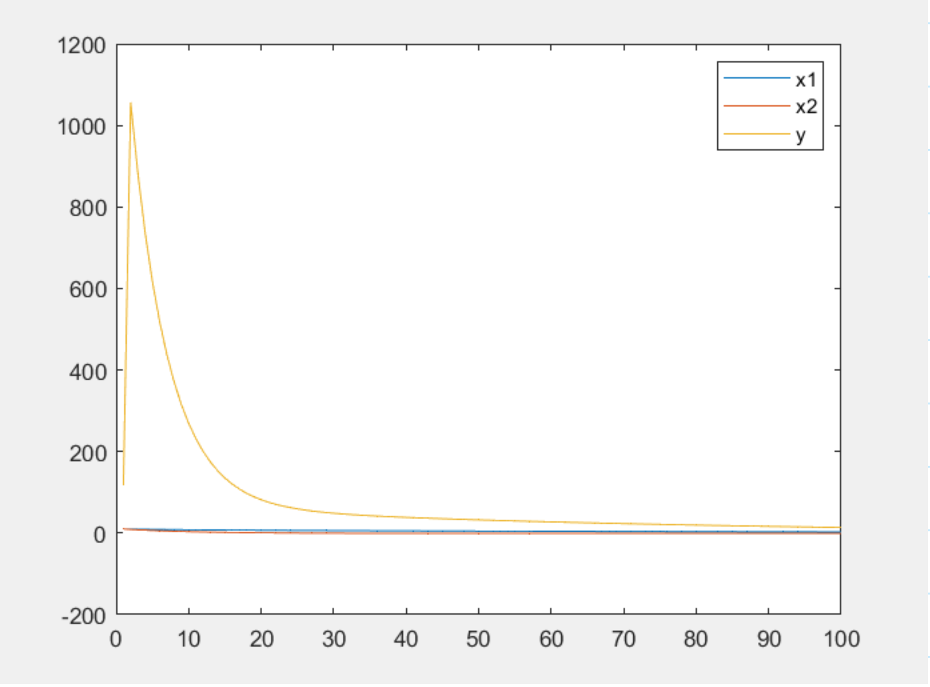
This is convergence plot with large Stepsize = 0.2. The primal variables already diverge.



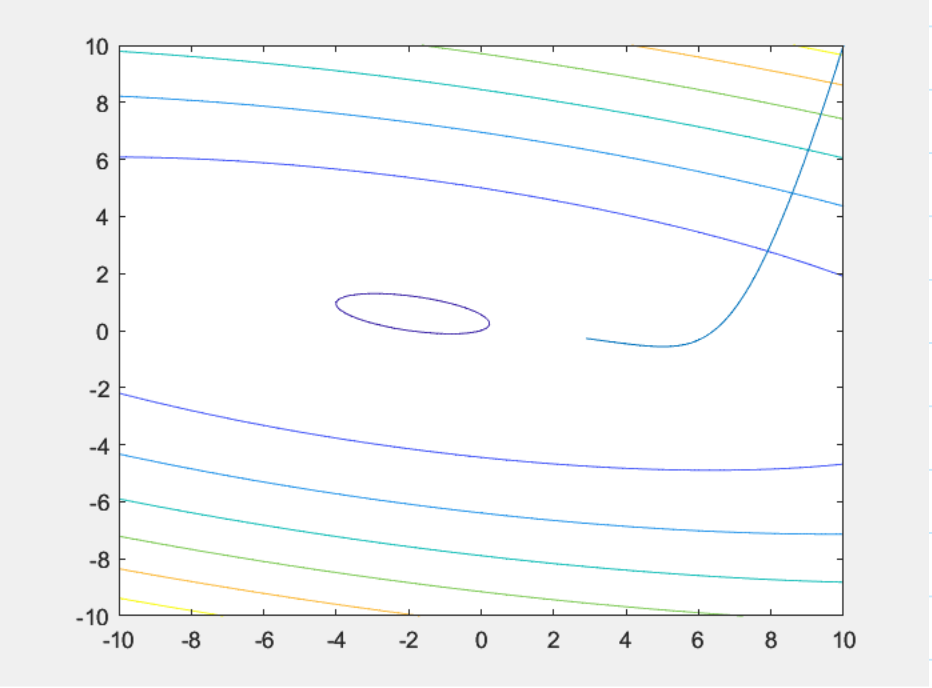
This is the contour plot, showing severe oscillation.



This is the convergence of primal variables with a small Stepsize = 0.01. Convergence is too slow.



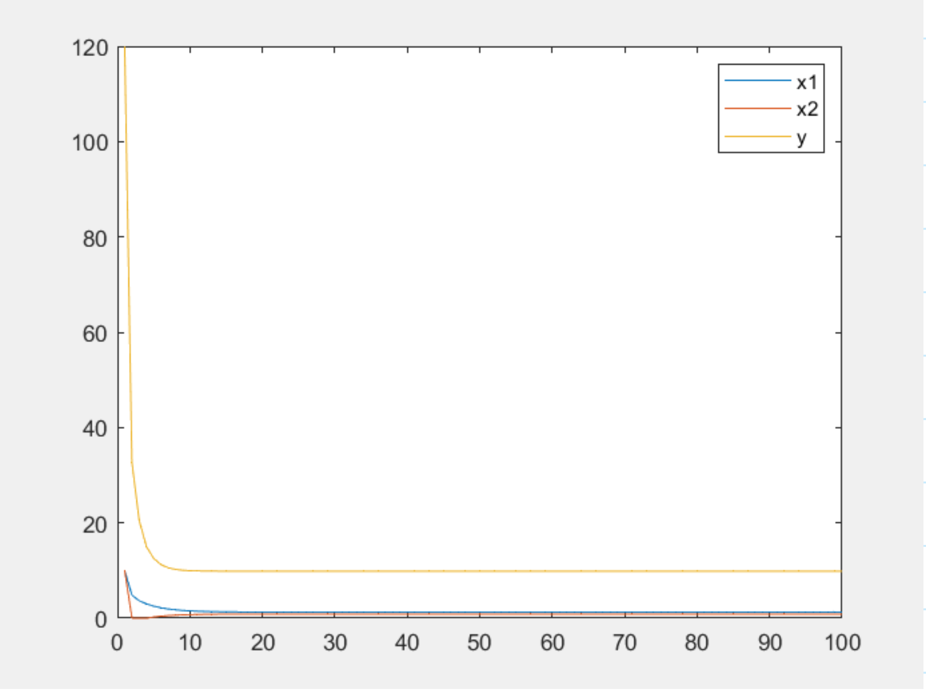
This the contour plot. The variables do not converge to the optimal point yet.


10
8
6
4
2
0
-2
-4
-6
-8
-10
-10 -8 -6 -4 -2 0 2 4 6 8 10

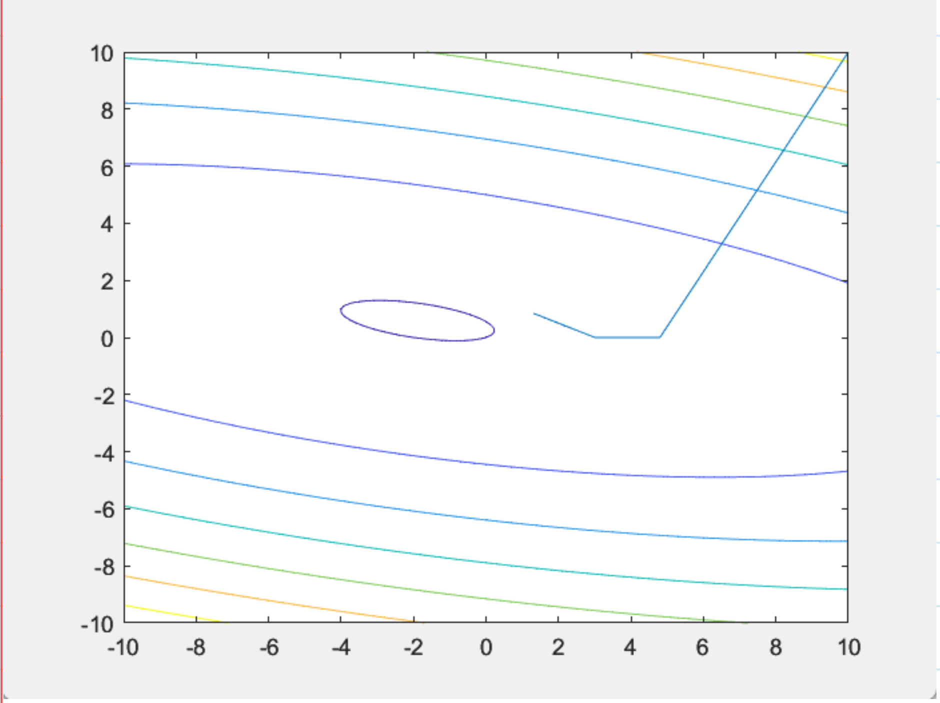
P1c

Thursday, March 30, 2023 1:45 PM

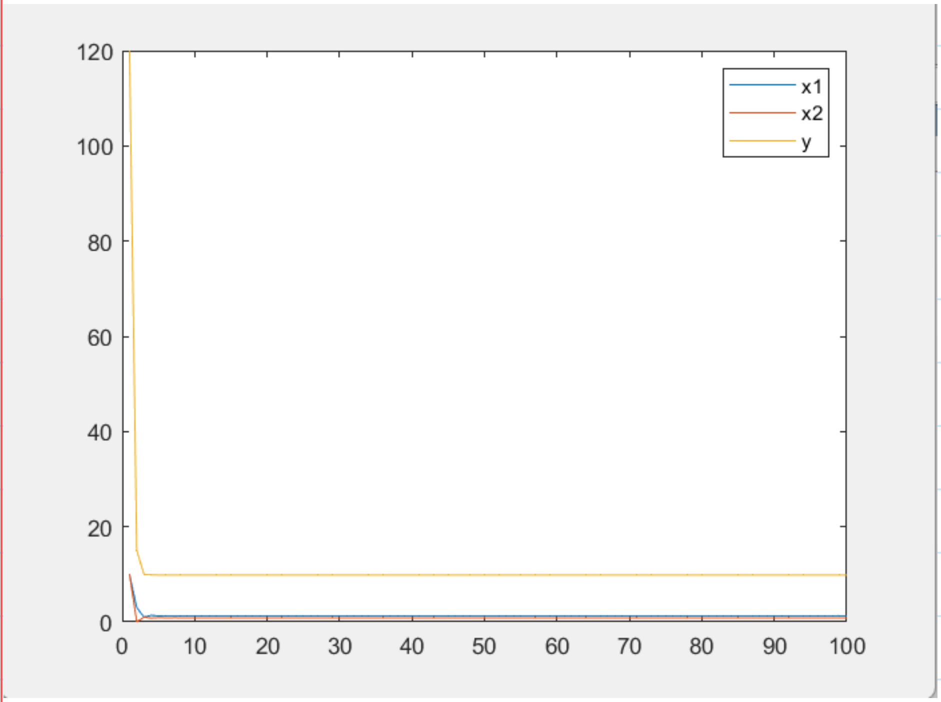
This is convergence plot for the primal variables with Stepsize = 0.1. Convergence speed is about right.
The final point is
x1=1.285714, x2=0.857143



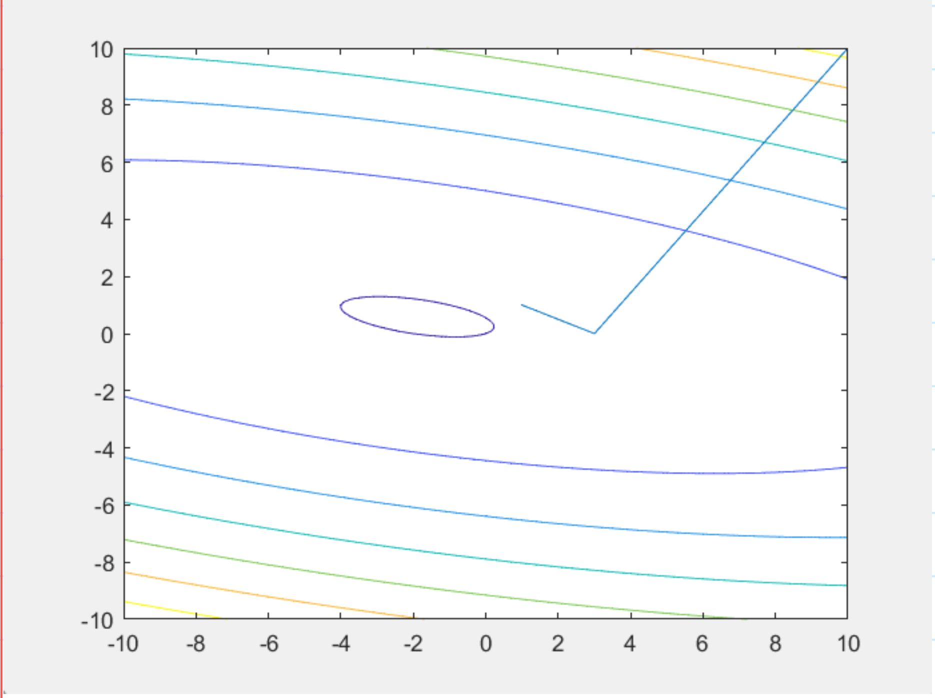
This is the contour plot.



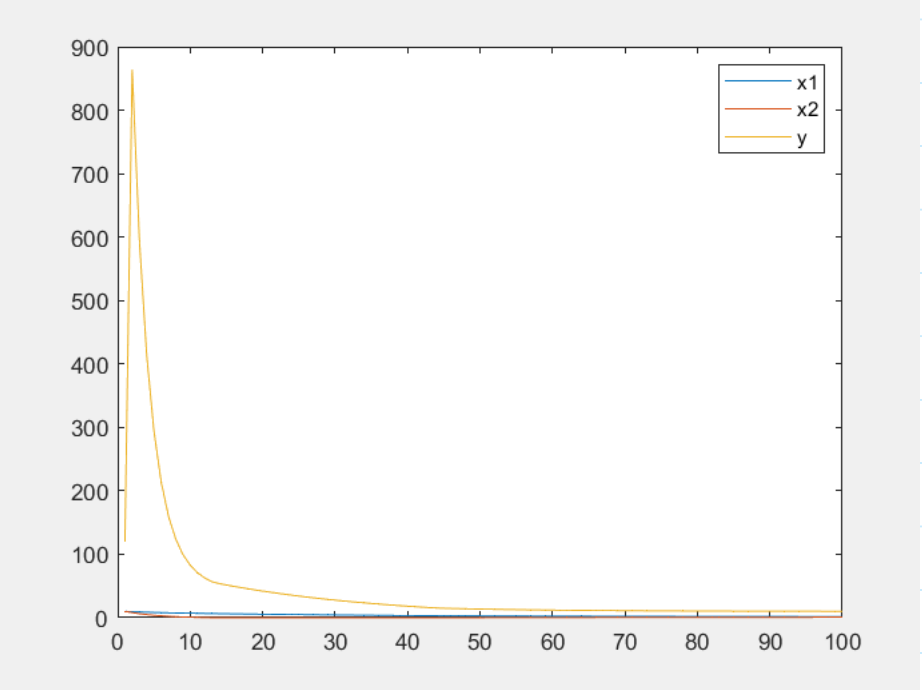
This is the convergence plot with a large Stepsize = 0.5. Surprisingly it still converge to a neighborhood of the optimal solution. The conditions that we studied in class for convergence are sufficient but not necessary, which leaves possibility like this.



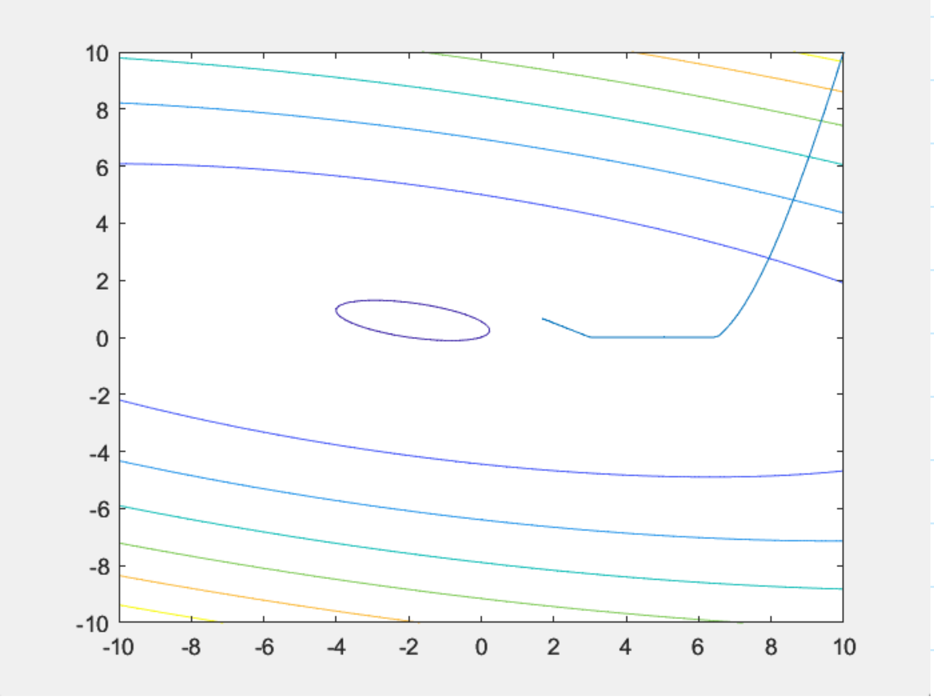
This is the contour plot.



This is the convergence plot with a small Stepsize = 0.01. Again the convergence is slow.



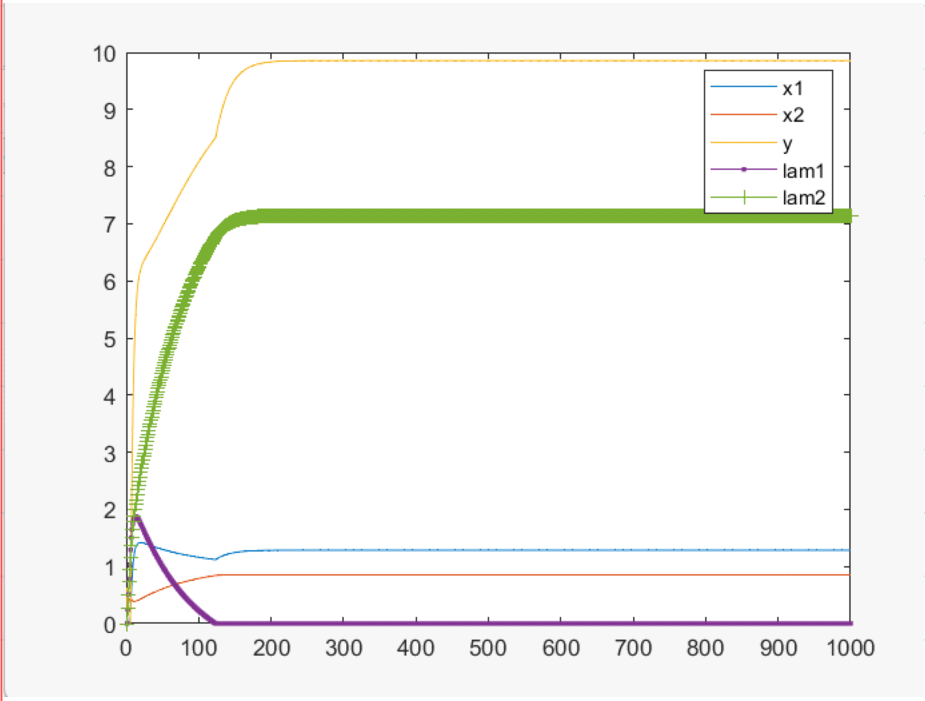
This is the contour plot.


10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

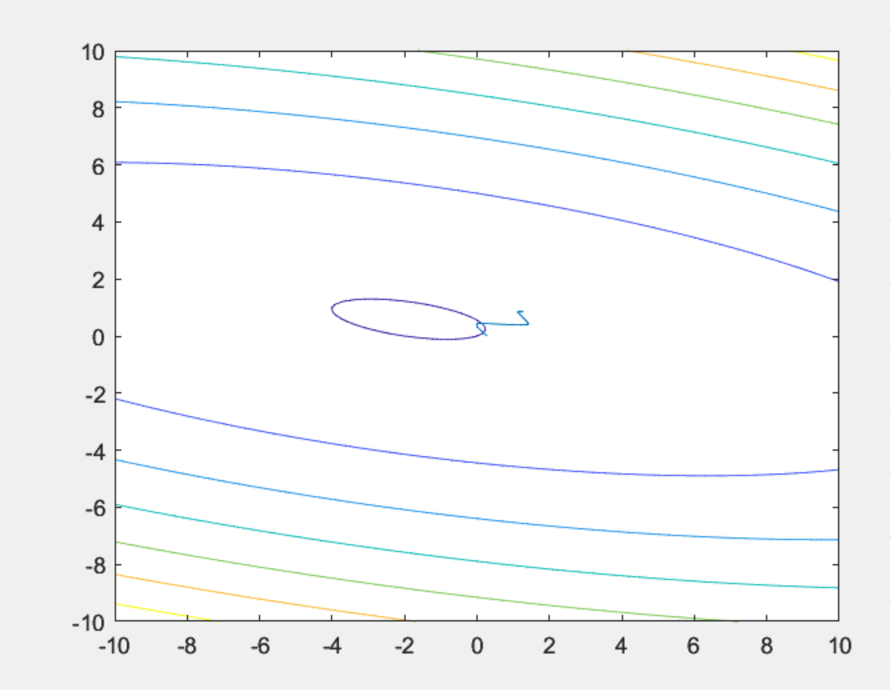
P1d

Thursday, March 30, 2023 1:48 PM

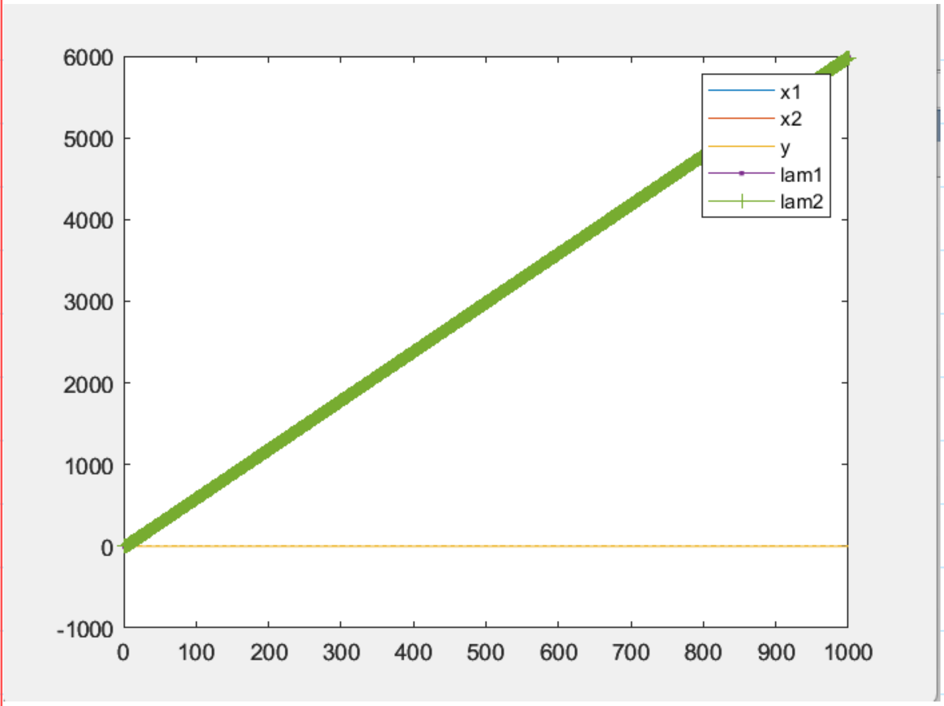
This is the convergence plot for both primal and dual variables with Stepsize = 0.1. Convergence speed is about right. The final values are
x1=1.285714, x2=0.857143
lam1=0.000000, lam2=7.142857



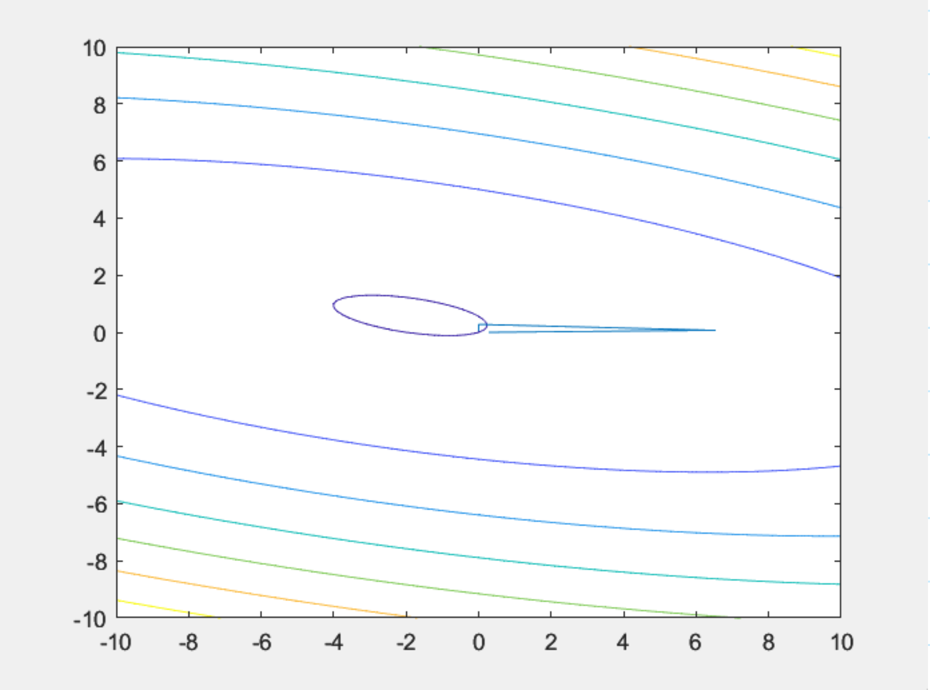
This is the contour plot for the primal variables.



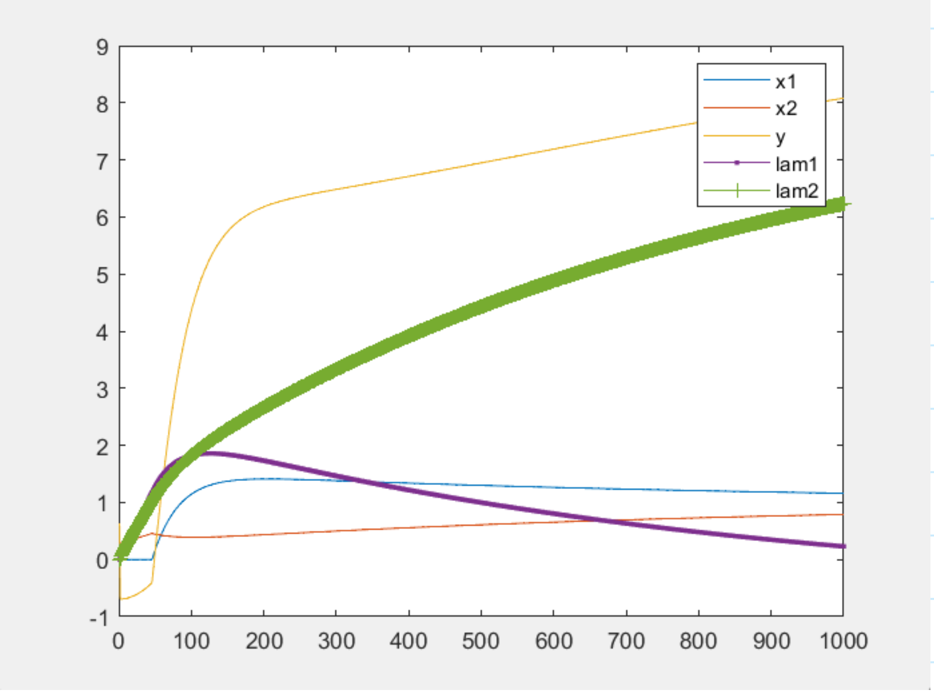
This is the convergence plot with a large Stepsize = 2.0. The dual variables diverge.



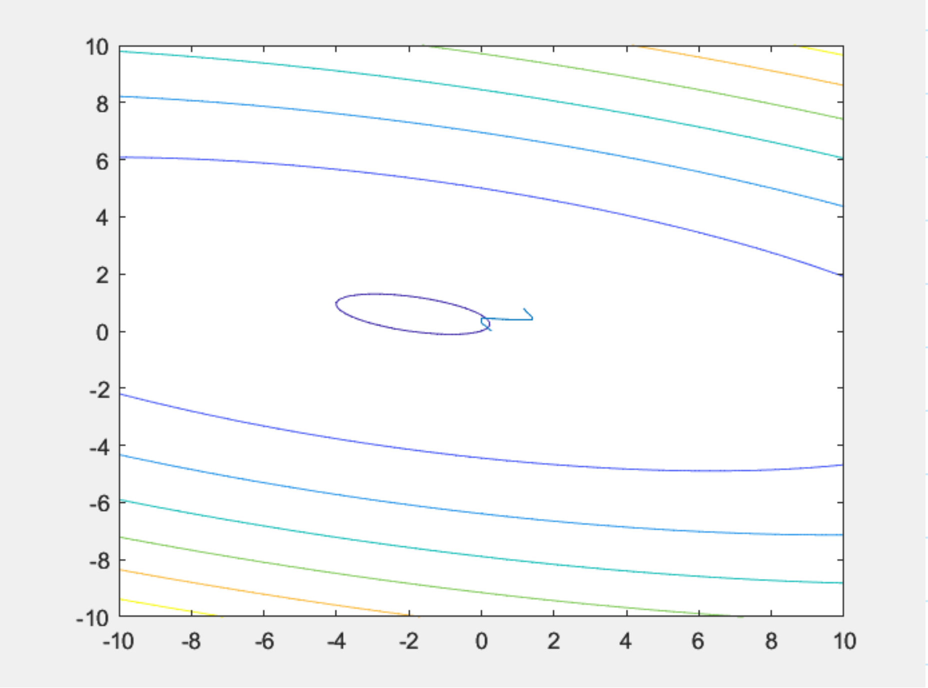
This is the contour plot:



This is the convergence plot with a small Stepsize = 0.01. Convergence is again very slow.



This is the contour plot.


10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

P1e

Thursday, March 30, 2023 1:54 PM

For Problem 1(b), we can check the gradient to be close to zero:

x1=-1.888888, x2=0.592590
gradient=-0.000004, -0.000036

For Problem 1(c), we first check which constraint is binding.

x1=1.285714, x2=0.857143
x1+2.0*x2=3.000000

We then check that the inner product between the gradient and the direction of the line above, i.e., (-2.0, 1.0), is zero, suggesting that the gradient is in the normal cone:

normality to the line above: 0.000000

For Problem 1(d), we can check the KKT condition. We first check the primal constraints:

x1=1.285714, x2=0.857143
x1+2.0*x2=3.000000

We also check the dual constraints:

lam1=0.000000, lam2=7.142857. Both >=0

From lam1=0.000000, we can see that the complementary slackness condition is satisfied.

Finally, we check that x1 and x2 minimizes the Lagrangian:

gradient of Lagrangian 0.000000, 0.000000

P2a

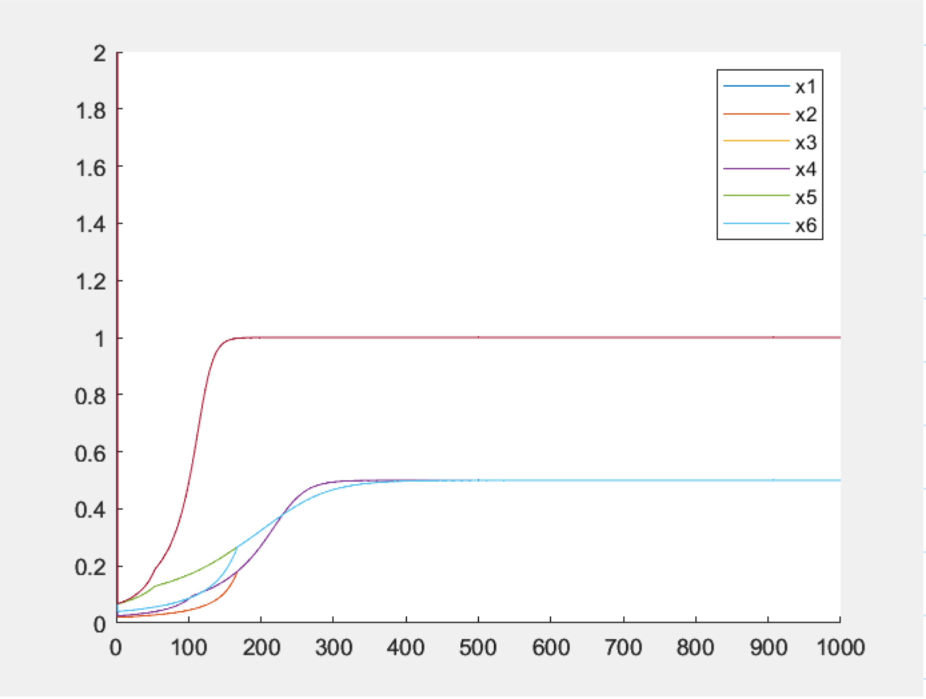
Thursday, March 30, 2023 2:00 PM

The dual algorithm can be designed as we did in class.

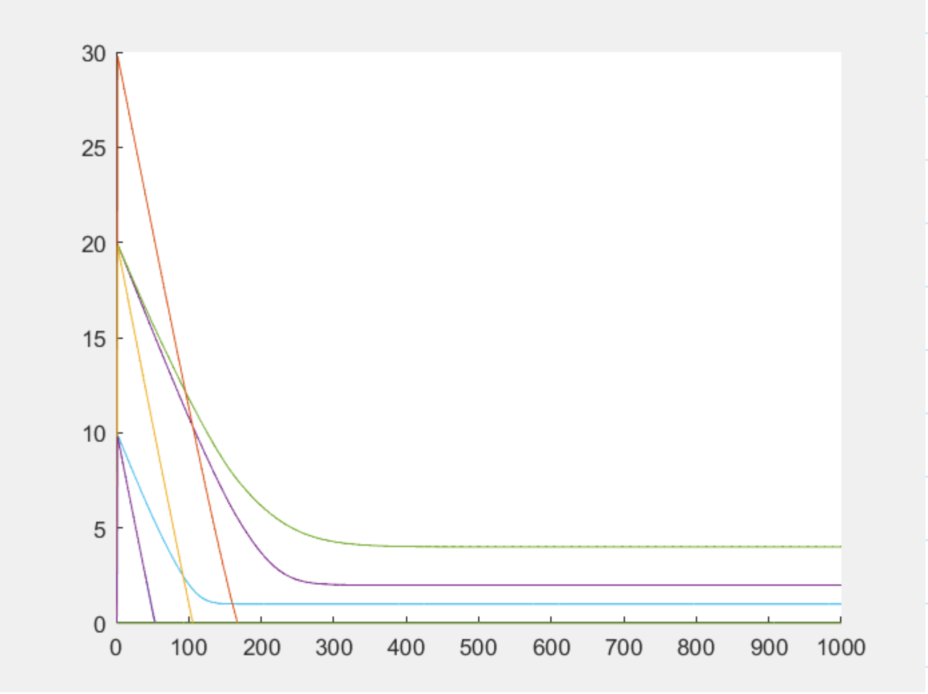
P2b

Thursday, March 30, 2023 2:00 PM

This is convergence plot for the flow rates.



This is the convergence plot for the link prices (i.e., dual variables)



30
25
20
15
10
5
0
0
100
200
300
400
500
600
700
800
900
1000

P2c

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We verify that the rate minimizes the Lagrangian. For each flow, the number on the LHS is the current rate, the number on the RHS is the one minimizing the Lagrangian:

Flow_Rate:
(#1)0.500000=0.500000
(#2)0.500000=0.500000
(#3)0.500000=0.500000
(#4)0.500000=0.500000
(#5)0.500000=0.500000
(#6)0.500000=0.500000
(#7)1.000000=1.000000

The link prices are all non-negative, and hence are dual feasible.

Below, for each link, we check whether the primal constraint is satisfied. For each link, the value to the left is the link price. The middle value is the current total rate on the link, the value to the right is the link capacity. If the dual price is positive, the corresponding constraint must be equal. Otherwise, it is <=. This is true for all links, hence verifying both the primal feasibility and the complementary slackness condition.

Link_Price:
(#1)1.000000, (1.000000=1.000000)
(#2)0.000000, (0.000000<1.000000)
(#3)2.000000, (1.000000=1.000000)
(#4)2.000000, (1.000000=1.000000)
(#5)4.000000, (1.000000=1.000000)
(#6)1.000000, (1.000000=1.000000)
(#7)0.000000, (0.000000<2.000000)
(#8)0.000000, (0.500000<2.000000)
(#9)0.000000, (1.500000<2.000000)
(#10)0.000000, (1.000000<2.000000)
(#11)0.000000, (1.000000<2.000000)
(#12)0.000000, (0.000000<2.000000)

Thus, the KKT condition is verified, and hence the solution is optimal.