Lec23-mwf

Monday, March 09, 2009 5:38 PM

Deriving AIMD window-based control

Wednesday, March 11, 2009 3:07 PM

- Take $\alpha = 1$

- Approximate the differential equation by a difference equation

$$\frac{x_s(t+\delta) - x_s(t)}{\delta} = K_s\left[\gamma_s - \beta x_s \cdot z_s\right)$$

- Recall that

$$x_s(t) = \frac{w_s(t)}{d_s}$$

window size

RTT

Let $N_s(t, t+\delta)$ denote the # of nacks (or time-outs) received by user s in the interval $(t, t+\delta)$.

Then $z_s \simeq \dfrac{N_s(t, t+\delta)}{\delta}$

- Hence

$$\frac{w_s(t+\delta) - w_s(t)}{\delta d_s} = K_s\left[\gamma_s - \beta \frac{w_s(t)}{d_s} \frac{N_s(t, t+\delta)}{\delta}\right]$$

$$\Rightarrow w_s(t+\delta) - w_s(t) = K_s\left[\gamma_s \delta d_s - \beta w_s(t) N_s(t, t+\delta)\right]$$

- Let $A_s(t, t+\delta)$ denote the # of acks received by user s in the time-interval $(t, t+\delta)$.

$$x_s(t) = \frac{W_s(t)}{d_s} \approx \frac{A_s(t, t+\delta)}{\delta}$$

– Using $\delta = \frac{A_s(t,t+\delta)}{x_s(t)} = \frac{A_s(t,t+\delta)}{W_s(t)} \cdot d_s$

$$\Rightarrow W_s(t+\delta) - W_s(t) = K_s\Big[\frac{r_s d_s^2}{W_s(t)} A_s(t,t+\delta) - \beta W_s(t) N_s(t,t+\delta)\Big]$$

Interpretation: If $K_s r_s d_s^2 = 1$ & $\beta K_s = \frac{1}{2}$

– For each ack, window-size increases by $\frac{1}{W_s}$

– For each nack, window-size decreases by $\frac{1}{2} W_s$

– AIMD

(30)

AIAD - skip

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- Take $U_s(x_s) = \gamma_s \log x_s$

- Approximate the differential equation by a difference equation

$$\frac{x_s(t+\delta) - x_s(t)}{\delta} = K_s[\gamma_s - \beta \cdot z_s]$$

- Recall that

$$x_s(t) = \frac{w_s(t)}{d_s}$$

window size

RTT

Let $N_s(t, t+\delta)$ denote the # of nacks (or time-outs) received by user s in the interval $(t, t+\delta)$.

Then $z_s \simeq \frac{N_s(t, t+\delta)}{\delta}$

- Hence

$$\frac{w_s(t+\delta) - w_s(t)}{\delta\, d_s} = K_s\left[\gamma_s - \beta \frac{N_s(t, t+\delta)}{\delta}\right]$$

$$\Rightarrow w_s(t+\delta) - w_s(t) = K_s[\gamma_s \delta d_s - \beta d_s N_s(t, t+\delta)]$$

- Let $A_s(t, t+\delta)$ denote the # of acks received by user s in the time-interval $(t, t+\delta)$.

$$X_s(t) \simeq \frac{A_s(t, t+\delta)}{\delta}$$

- Using $\delta = \dfrac{A_s(t,t+\delta)}{X_s(t)} = \dfrac{A_s(t,t+\delta)}{W_s(t)} \cdot d_s$

$$\Rightarrow W_s(t+\delta) - W_s(t) = K_s\Big[\frac{r_s d_s^2}{W_s(t)} A_s(t,t+\delta) - \beta d_s N_s(t,t+\delta)\Big]$$

Interpretation: If $K_s r_s d_s^2 = 1$ & $\beta K_s d_s = 1$,

- For each ack, window-size increases by $\frac{1}{W_s}$
- For each nack, window-size decreases by 1.
- AIAD

Packet loss rate

Wednesday, March 11, 2009 4:22 PM

- One consequence of this interpretation is that we can derive an equation that relates the TCP throughput with the packet loss rate.

- Recall the differential equation

$$\dot{x}_s = k_s(\alpha \gamma_s - \beta x_s^{\alpha} \cdot z_s)$$

- Using the parameters corresponding to AIMD:

$$\alpha = 1, \quad k_s \gamma_s = \frac{1}{d_s^2}, \quad k_s \beta = 0.5,$$

$$\Rightarrow \dot{x}_s = \left(\frac{1}{d_s^2} - 0.5 x_s \cdot z_s \right)$$

At equilibrium, $\dot{x}_s = 0$, we then have

$$\frac{1}{d_s^2} - 0.5 x_s \cdot x_s \cdot p_s = 0$$

where p_s is the packet-loss probability experienced by user s. We then have

$$x_s = \frac{1}{d_s \sqrt{\frac{p_s}{2}}}$$

Interpretation:

① TCP throughput is inversely proportional to RTT.

— long flows suffer

② TCP throughput is inversely proportional to the square root of Ps

Both have been observed empirically.

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Critique

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① Penalty function approach does not produce exact solution to the original problem.
   - packet drops will occur at equilibrium

② when buffer size is large, delay will increases

   - TCP will always drive the buffer full.

Later we will investigate other approaches:

- duality, primal-dual
- AQM: RED, REM, AVQ.

Duality approach to congestion control

Thursday, March 19, 2009 4:45 PM

- We now want to develop other solutions to the congestion control problem

$$\max \quad \sum_s U_s(x_s)$$
$$\text{sub to} \quad \sum_s H_s^l x_s \le R_l \quad \forall l \qquad q_l$$
$$m_s \le x_s \le M_s$$

- Note that this formulation incorporates a number of objectives
  - high utilization
  - low congestion
  - fairness

- We want our new solution to also retain the distributed nature of TCP
  - source only needs a congestion signal from the links that it passes through (e.g. packet loss)
    - no information of other user's utility
  - link only needs the aggregate rate
    - no information of each individual user.
    - no per-flow information.

Now let us see if duality helps.

Associate a Lagrangian multiplier q_l for each link.
The Lagrangian is given by

$$L(\vec{x}, \vec{q}) = -\sum_s U_s(x_s) + \sum_l q_l \left[\sum_s H_s^l x_s - R_l\right]$$

$$= -\sum_s \left[U_s(x_s) - x_s \sum_l q_l H_s^l\right] - \sum_l q_l R_l$$

separable.

The dual objective function is given by

$$g(\vec{q}) = \min_{\vec{x}} L(\vec{x}, \vec{q})$$

$$= -\sum_s \max_{x_s} \left[U_s(x_s) - x_s \sum_l q_l H_s^l\right] - \sum_l q_l R_l$$

Hence, each user should maximize

$$U_s(x_s) - x_s \cdot \sum_l q_l H_s^l$$

$$\Rightarrow \quad U_s'(x_s) = \sum_l q_l H_s^l$$

$$x_s^* = \left[U_s'^{-1}\left(\sum_l q_l H_s^l\right)\right]^+_{[m_s, M_s]}$$

Further,

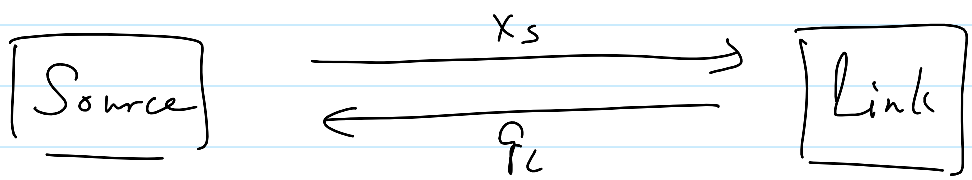
$$\frac{\partial}{\partial q_l} g = \sum_s H_s^l x_s - R_l$$

In order to maximize g

$$q_l(t+1) = \left[q_l(t) + \gamma\left(\sum_l H_s^l x_s^*(t) - R_l\right)\right]^+$$

To summarize:

- The dual variable q_l can be viewed as the "price" of capacity at link l
- Source algorithm: Knowing the price of all links passed by the user, each user computes $x_s(t)$ that maximizes the net utility $U_s(x_s) - x_s \sum_l q_l H_s^l$
- Link algorithm: Each link update q_l to balance supply & demand.



Optimality of dual congestion controller - skip

Wednesday, March 25, 2009 9:36 AM

Theorem: Assume that

① On the interval $[m_s, M_s]$, the utility function U_s is increasing, strictly concave, and twice continuously differentiable

② For feasibility, assume $\sum_s H_s^l m_s < R^l \ \forall l$

③ The curvature of $U_s(\cdot)$ is bounded away from zero on $[m_s, M_s]$, i.e.,

$$-U_s''(x_s) \geq \frac{1}{\alpha_s} \quad \text{for all } x_s \in [m_s, M_s].$$

If the stepsize γ satisfies

$$0 < \gamma < \frac{2}{\bar{\alpha} \bar{L} \bar{S}},$$

where $\bar{\alpha} = \max_s \alpha_s$, $\bar{L} = \max_s \sum_l H_s^l$ is the length of the longest-path used by any user, and $\bar{S} = \max_l \sum_s H_s^l$ is the # of users sharing the most-congested link, then starting from any $\vec{q}(0)$, the sequence $(x(t), q(t))$ converges, and the limit is primal-dual optimal.

Note: Assume perfect feedback of q_l.

Proof: Show that the dual objective function satisfies a Lipschitz condition. (see hand-out window-inverse.pdf).

Note that $\frac{\partial}{\partial q_l} g = \sum_s H_s^l x_s^* - R_l$. Let $\vec{q}^1, \vec{q}^2$ be two price vectors, and let $x_s(\vec{q}^1)$ & $x_s(\vec{q}^2)$ be the corresponding rate control decisions (that maximizes net-utility). If g satisfies a Lipschitz condition, i.e

$$\|\nabla g(\vec{q}^1) - \nabla g(\vec{q}^2)\| \le \beta \|\vec{q}^1 - \vec{q}^2\|$$

which implies

$$-\left(\nabla g(\vec{q}^1) - \nabla g(\vec{q}^2)\right)\cdot\left(\vec{q}^1 - \vec{q}^2\right)$$
$$\le \frac{1}{\beta} \|\nabla g(\vec{q}^1) - \nabla g(\vec{q}_2)\|^2$$

The latter is equivalent to.

$$- \sum_l \left[\left(\sum_s H_s^l x_s(\vec{q}^1) - R_l\right) - \left(\sum_s H_s^l x_s(\vec{q}^2) - R_l\right)\right] \times \left(q_l^1 - q_l^2\right)$$

$$\ge \frac{1}{\beta} \sum_l \left[\left(\sum_s H_s^l x_s(\vec{q}^1) - R_l\right) - \left(\sum_s H_s^l x_s(\vec{q}^2) - R_l\right)\right]^2$$

See hand out for the detailed proof to show that $\beta = \bar{L} \bar{S} \bar{L}$

Refs:

- S. H. Low and D. E. Lapsley, "Optimization Flow Control-I: Basic Algorithm and Convergence," IEEE/ACM Transactions on Networking, vol. 7, no. 6, pp. 861-874, December 1999.
- S. Low, "A Duality Model of TCP and Queue Management Algorithms," IEEE/ACM Trans. on Networking, 11(4):525-536, August 2003

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