

# Lec22

Saturday, March 4, 2023 10:59 AM

Bring handout

Mid-term Exam:

1. Midterm: 11/7 Thursday 8am: available on Blackboard
  - a. Due next day at 12:00pm.
  - b. Take 2-4 hours
2. Take home. No discussion with others. Can consult book or paper.
3. Coverage up to lectures 1-24.
  - a. Convex sets
  - b. Convex functions
  - c. Convex programs and optimal solutions
  - d. Duality, KKT conditions, duality gap.
  - e. Optimization algorithms, convergence.
  - f. Some application: background info will be provided
4. Sample exam on the web.
5. Review notes and homework problems!

- IP is best effort
- TCP provides reliability on top of IP
- Window-based control increases throughput but also congestion
- TCP window adjustment:
  - Congestion avoidance
  - Three goals
  - Desirable features

# Congestion control

Sunday, March 08, 2009 4:00 PM

So far we have discussed

- formulation of optimization problems into convex programs
- Optimality conditions
- Duality
- Optimization algorithms with practical interpretations

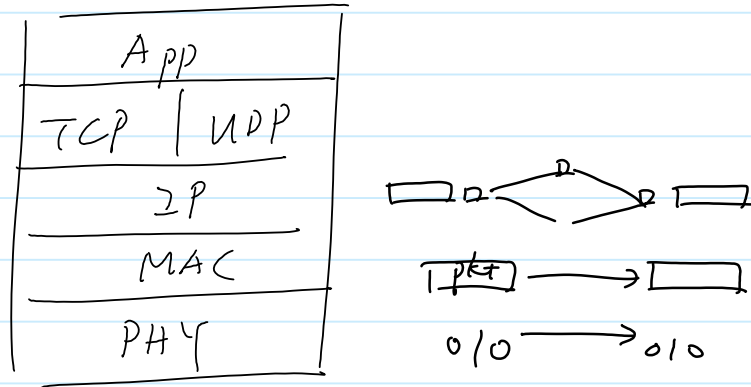
Next, we are going to use congestion control & TCP as practical examples of how to use convex optimization to design protocols

- which will introduce additional issues
  - delayed feedback
  - noise/randomness

# TCP

Sunday, March 08, 2009 4:22 PM

- The TCP/IP protocol stack



- Key summary:

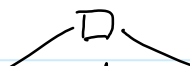
- IP is best effort
- TCP provides reliability on top of IP
- Window-based control increases throughput but also congestion
- TCP window adjustment:
  - Congestion avoidance
  - Three goals
  - Desirable features

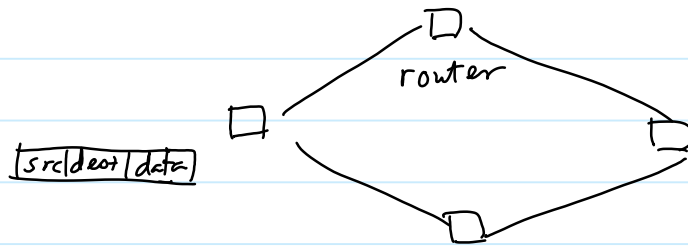
- IP provides best-effort packet delivery service over heterogeneous networks

- each computer has an IP address

- each data packet contains both the source IP addr & the destination IP addr.

- routers will route IP packets from the source to destination





- Best effort:
  - packet may be lost, duplicated, form loops or out-of-sequenced.
  - no guarantee.

#### Skip UDP

- UDP: just add a port number in the packet header
  - (port + addr) identifies the specific application on a specific computer
  - no improvement in service quality.
- TCP: provides not only a port number for each application, but also a connection-oriented, reliable, in-order packet delivery service over IP.
  - connection-oriented and in-order: both end-points maintain the "state" of the connection
  - open / close: an OS call binds (src IP, , , , dest IP, , , , port, , , , port)

' src port) to (dest IP, dest port)

- Sequence number:

- source maintains seq # for the next byte to be sent

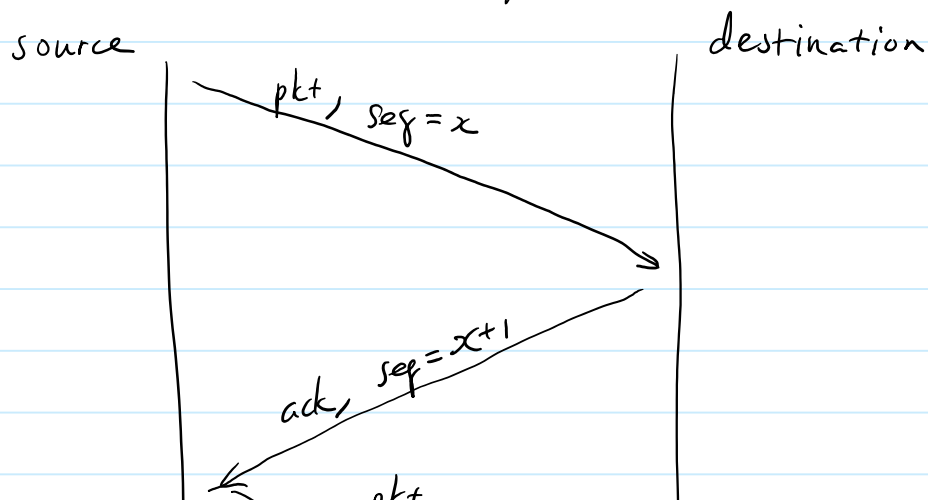
- receiver maintains seq # for the next byte expected

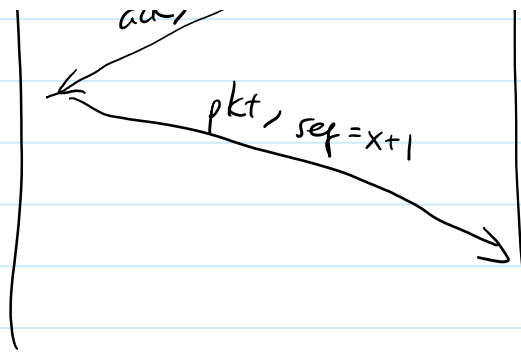
- The two sync the seq # at the beginning when the connection is opened.

- reliable:

- The seq # advances only after the packet is successfully acknowledged.

- If the packet is not acked within a time-out period (on the order of a round-trip-time RTT), the source will retransmit the packet.



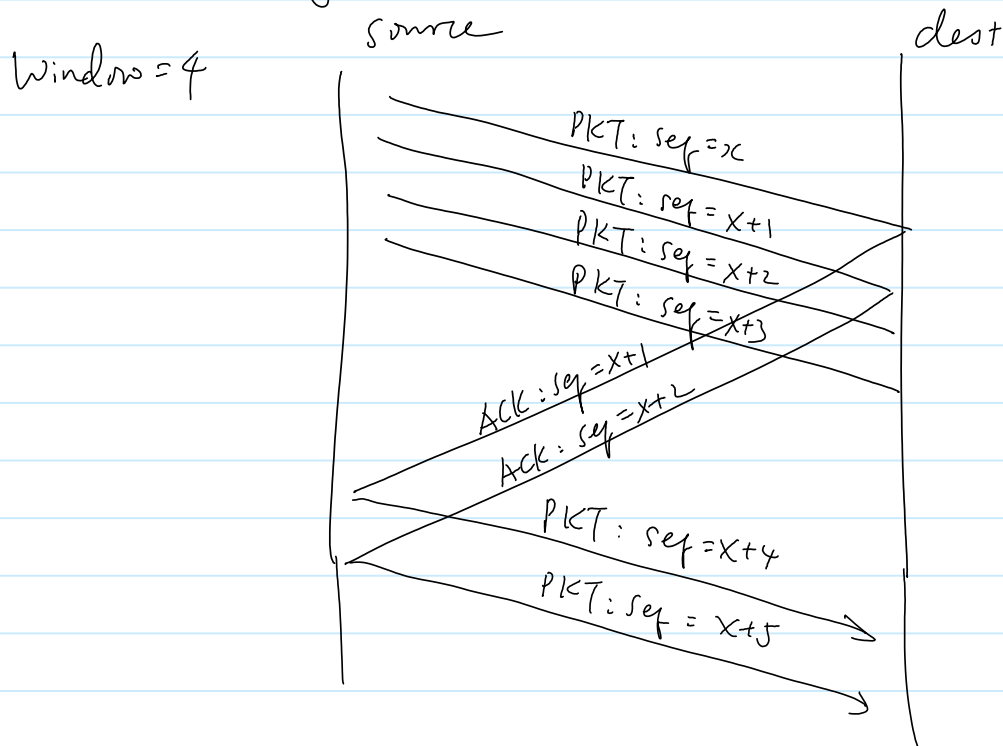


- While this is reliable, it is too slow
- 1 packet every round-trip-time (RTT)

# Windows

Wednesday, October 25, 2023 4:36 PM

- To increase throughput, the source can send multiple packets into the network before waiting for an ack.
- Window: the # of outstanding packets that the source can send before waiting for an acknowledgement



- Clearly, the rate that packets are sent into the network increases with the window-size

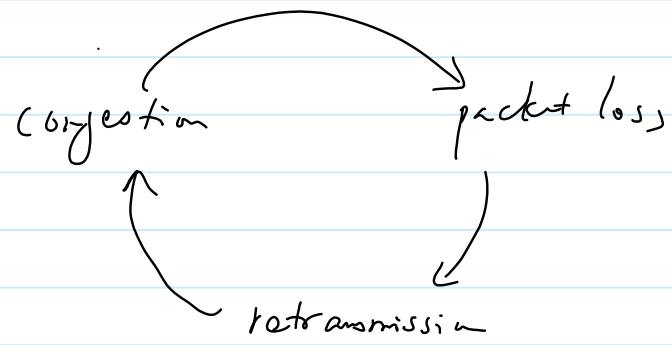
the network increases with the window-size

- If the window-size is too small, the network is under-utilized.
- If the window-size is too large, the network will become congested.
- In reality, when intermediate routers can not accommodate these packets, the packets are dropped, which leads to time-out & retransmission of the packet at the source
- If not controlled properly, the retransmission may further aggravated the congestion, which then leads to the so-called "congestion collapse."



Congestion collapse: no one is able to send any useful packets. packets keep being dropped & retransmitted.





## TCP window control

Wednesday, March 11, 2009 2:38 PM

① How does TCP control the window sizes.

skip - flow control: the receiver will advertise a window-size that corresponds to the # of outstanding packets that it can accommodate.

(This is due to a finite receiver buffer and that the application may not pick up packets fast enough.)

- flow control puts an upper bound on the window size.

- Congestion control: our main focus

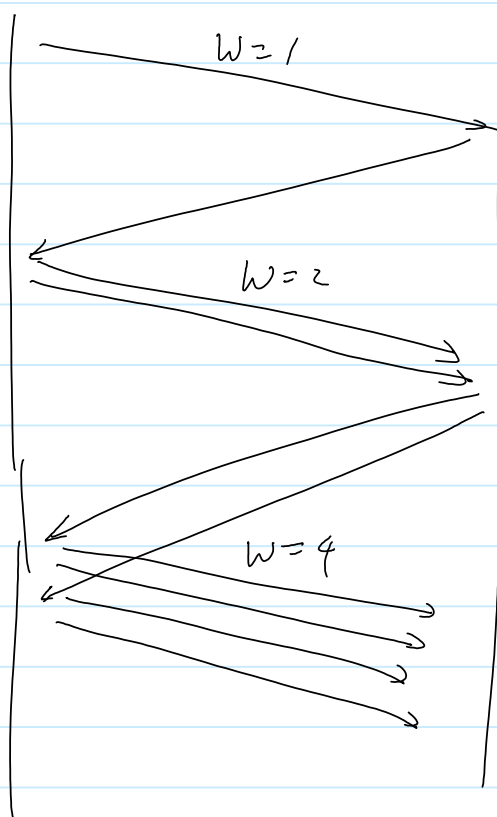
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Two phases:

skip

① "Slow start", which is not slow

- Used when starting a connection.
- Increase window by 1 at each ack
- Doubles the window size each RTT.



- "Slow start" stops when the window size exceeds the slow start threshold, or when packets are time-out/lost.

② "Congestion avoidance": Our focus

- On each new ack,  $cwnd$  increased by  $\frac{1}{cwnd}$
- After one RTT,  $cwnd$  increased by 1
- On each time-out,  $cwnd$  cut in half
- Known as AIMD (Additive Increase/Multiplicative Decrease)

- Believed to achieve three goals:

- to fully utilize the available bandwidth,
- to prevent the onset of congestion
- to be fair to different flows.

- Some highly desirable features of TCP

- Only need to be deployed at the end-points
- Very little support from routers (just drop packets when congested).

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- The above basic principles of TCP was published in Van Jacobson's 1988 paper after the first congestion collapse

- They forms the basis of TCP today.

- Many researchers have tried to understand the behavior of TCP

- Easier if there is only one link

- Utilization
- Congestion avoidance
- Fairness

- However, the case of multiple links is hard to track.
- Until Kelly's 1998 paper, which view congestion control as the solution of an optimization problem.

## Formulation

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- Kelly proposed to formulate congestion control as the following optimization problem, and view TCP as an iterative solution to this problem:

$$\max \sum_s U_s(x_s)$$

$$\text{sub to } \sum_s H_s^l x_s \leq R^l \quad \forall l$$

- $x_s$ : rate of user  $s$
  - $U_s(\cdot)$ : utility function
  - $H_s^l = 1$ : if user  $s$  uses link  $l$   
 $= 0$ : o/w
  - $R^l$ : capacity of link  $l$ .
- Note that all three objectives are incorporated
    - High utilization: using increasing  $U_s(\cdot)$
    - Congestion avoidance
    - Fairness: by choosing the utility function

eg: -  $U_s(x_s) = \log x_s$

$$\Rightarrow \sum_s \frac{x_s - x_s^*}{x_s^*} \leq 0$$

proportional fair

$$- u_s(x_s) = - \frac{x_s^{1-\alpha}}{1-\alpha}, \alpha > 0$$

- ① Is TCP solving such a problem?
- ② If yes, what utility function is TCP implicit w.r.t.?
- ③ Are there other ways to develop protocols that also solve such an optimization problem?
- ④ Can we work with more complex settings, e.g. cross-layer control?

### References:

- ① "End-to-end Congestion Control Schemes: Utility Functions, Random Losses and ECN Marks," S. Kunniyur and R. Srikant, IEEE/ACM Transactions on Networking, Vol. 11, No. 5, October 2003
- ② F. P. Kelly, A. K. Maulloo, and D. K. H. Tan, "Rate control for Communication Networks; Shadow prices, proportional fairness and stability", Journal of Operation Research Society, Vol. 49, No. 3, pp 237-252, March 1998.

## Primal solution: penalty approach

Wednesday, March 11, 2009 2:57 PM

- Consider the following problem:

$$\max_{\{x_s\}} \sum_s U_s(x_s) - \beta \cdot \sum_l \int_0^{\sum_s H_s^l x_s} P_l(R_l, x) dx$$

where the function  $P_l(R_l, x)$  is chosen to penalize the objective when  $\sum_s H_s^l x_s \geq R_l$ .

$$P_l(R, x) = \begin{cases} 0 & x \leq R \\ > 0 & x > R \end{cases}$$

e.g.  $P_l(R, x) = \frac{(x-R)^+}{x}$

- We will later interpret  $P_l$  as the packet dropping prob. at link  $l$ , when the packet incoming rate is  $x$ .
  - This new problem can be viewed as a penalty function approach to solve the original constrained problem.
- 

- Let  $f(\vec{x})$  denote the objective function minus penalty.



- The gradient is given by

$$\frac{\partial f}{\partial x_s} = U'_s(x_s) - \beta \cdot \sum_l H_s^l P_l(R_l, \sum_i H_s^l x_i)$$

- Each user can employ the following gradient-descent iteration.

$$x_s = k_s \left[ U'_s(x_s) - \beta \cdot \sum_l H_s^l P_l(R_l, \sum_i H_s^l x_i) \right]$$


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Example:

① If  $U_s(x_s) = \delta_s \cdot \log x_s$ , then

skip

$$x'_s = k_s \left[ \frac{\delta_s}{x_s} - \beta \cdot \sum_l H_s^l P_l(R_l, \sum_i H_s^l x_i) \right]$$

or

$$x'_s = k_s \left[ \delta_s - \beta x_s \cdot \sum_l H_s^l P_l(R_l, \sum_i H_s^l x_i) \right]$$

- Recall that  $P_l$  is the packet dropping prob. at link  $l$

- Then,  $\sum_l H_s^l P_l$  is approximately the packet dropping prob. along the path of user  $s$ .

- Let  $\zeta_s = x_s \sum_l H_s^l P_l(R_l, \sum_i H_s^l x_i)$  which represents the rate of packets that are lost. Then

are lost. Then

$$\dot{x}_s = K_s (\gamma_s - \beta z_s)$$

- A2AD

② If  $U_s(x_s) = -\frac{\gamma_s}{x_s^\alpha}$

$$\dot{x}_s = K_s \left( \frac{\alpha \gamma_s}{x_s^{\alpha+1}} - \beta \cdot \sum_l H_s^l P_L(R_L, \sum_l H_s^l x_s) \right)$$

- Recall that  $P_L$  is the packet dropping prob. at link  $l$

- Then,  $\sum_l H_s^l P_L$  is approximately the packet dropping prob. along the path of user  $s$ .

- Let  $z_s = x_s \sum_l H_s^l P_L(R_L, \sum_l H_s^l x_s)$  which represents the rate of packets that are lost. Then

$$\dot{x}_s = \tilde{K}_s (\alpha \gamma_s - \beta \cdot x_s^\alpha \cdot z_s)$$

If  $\alpha=1$ , the latter represents A2MD.

②b