Lec11-new

Thursday, February 03, 2011 9:47 AM

- We have seen that, once we formulate a convex optimization problem, we can write down precise conditions (both sufficient and necessary) for its optimal solution

- Later on we will further study effective algorithm to solve the optimal solution

- However, often the challenge in research is to obtain the convex problem first

- Below, we will use the resource sharing problem as an example, and gain some experience in formulating convex optimization problems in increasingly complex settings.

Rate allocation of the Internet: multiple resources

Saturday, January 31, 2009 5:21 PM

Single-Path:

- R^l : the capacity of link l
- x_s : the rate allocated to user s
- $U_s(x_s)$: the utility to user s.
- $[H_s^l]$: routing matrix

$$H_s^l = \begin{cases} 1 & \text{if user } s \text{ uses link } l \\ 0 & \text{otherwise} \end{cases}$$

$\sum_s H_s^l x_s$: total amount of traffic on link l.

Q How to allocate the rates?

$$\max \sum_s U_s(x_s)$$

$$\text{sub to } \sum_s H_s^l x_s \le R^l \quad \text{for all } l.$$

- A convex problem if $U_s(\cdot)$ is concave
- Physical meaning: Congestion Control
  - High-throughput
  - Avoid-congestion
  - Fairness (related to utility)

- Fairness (related to utility function)

- Wait until duality for the optimality condition.

Ref: J. Mo & J. Walrand, Fair end-to-end Window-based Congestion Control, IEEE/ACM Transactions on Networking, Vol. 8. No. 5, pp 556-567, Oct 2000.

Multipath congestion control

Saturday, January 31, 2009 5:34 PM

- Jointly routing & congestion control.
- Let each user has $O(s)$ paths

  $H_{sj}^{l} = 1$ if path j of user s uses link l
  $\phantom{H_{sj}^{l} = }\, 0$ otherwise

- Let x_{sj} be the data rate of user s on path j.
  - Constraint becomes $\sum_{s,j} H_{sj}^{l} x_{sj} \le R_l$ (*)
    for all l

- $\max \sum_s U_s\left(\sum_j x_{sj}\right)$

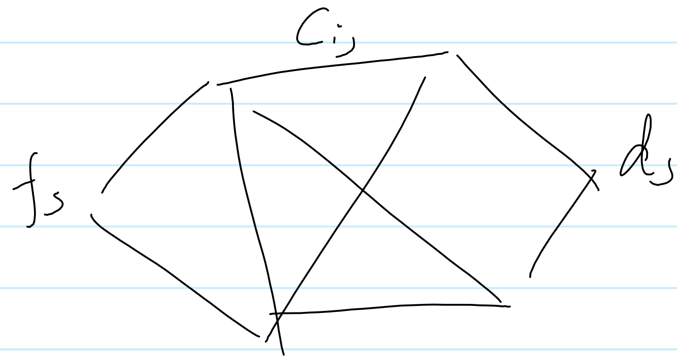
  sub to (*) for all l.

- Still a convex problem

What if routes are not given before-hand?

Routing

Sunday, January 25, 2009 11:45 AM



- S users

- Each user $s = 1, 2, \cdots, S$ sends a flow from node f_s to d_s, at the rate of x_s

- L : set of links

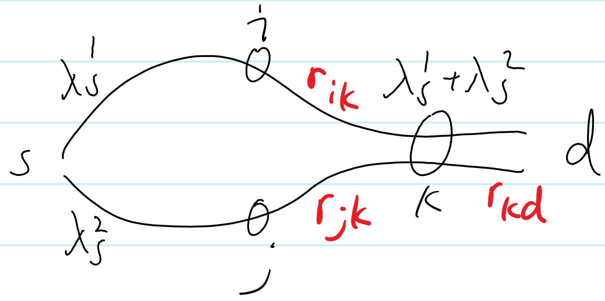
  - each link (i, j) from node i to node j

  - capacity of link (i, j) is c_{ij}

Q How to route the flows?

Basic Node-Balance Equation

- Let r_{ij}^s denote the amount of capacity on link (i,j) that is allocated to flows



- Then any feasible flow (routing) is equivalent to $[r_{ij}^s]$ that satisfies the following node balance equation:

For any node i:

total out-going flow at node i

$$(*) \quad \sum_{j:(i,j)\in L} r_{ij}^s + x_s 1_{\{i=d_s\}} = \sum_{j:(j,i)\in L} r_{ji}^s + x_s 1_{\{i=f_s\}}$$

total incoming flow at node i.

Capacity Constraints

$$(**) \quad \sum_s r_{ij}^s \le C_{ij}$$ (If link capacity on each direction is separate)

or $\sum_s (r_{ij}^s + r_{ji}^s) \le C_{ij} = C_{ji}$ (If a whole bi-direction capacity is defined)

Objectives

(a) Maximize utility

$$\max \sum_s U_s(x_s)$$

sub to (*) & (**)

Other possibilities: skip

(b) Just feasibility

$$\min \quad 0$$

sub to (*) and (**)

(c) Maximize future throughput (in proportion to a reference λ_s)

- Find the largest α such that the rate of each flow can be increased to $\alpha \lambda_s$.

$$\max \quad \alpha$$

sub to $\quad x_s = \alpha \lambda_s \quad \forall s$

(*) and (**)

- When there is only one commodity, it reduces to the max-flow problem.

(d) Minimize congestion at fixed $x_s = \lambda_s$

- Define a congestion measure for each link $\beta_{ij}\left(\sum_s r_{ij}^s\right)$

- Minimize the total congestion level of the network.

- homework : formulate convex problems

Ref: Ch 5. Bertsekas & Tsitsiklis
Parallel & Distributed Computation: Numerical Methods.

(30)

Cross-layer

Saturday, January 31, 2009 5:37 PM

So far, all these examples easily lead to convex problems

- The capacity of each link is assumed to be fixed

We will see that this changes quickly when we allow the link rate to be a function of other control variables

- This is common in wireless networks
- The link rate will depend on
  - Transmission power
  - Transmission schedule

We may still formulate these control decisions into a unified optimization problem

→ "Cross-layer control"

- rate-control → Transport layer
- routing → network layer
- link scheduling → MAC layer
- Power control → Physical layer

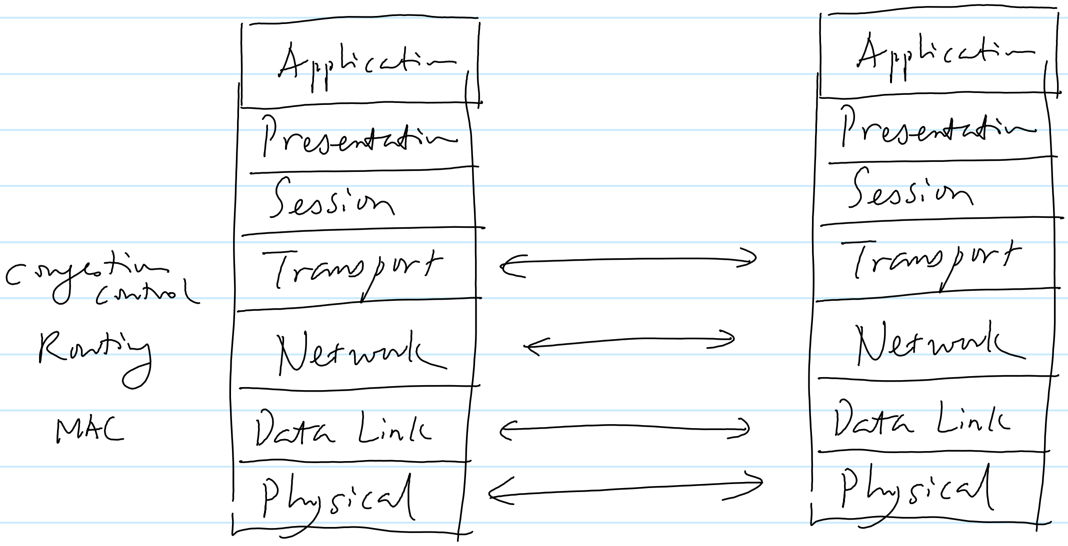
Q Why do we want to consider multiple

layers together?

- In Wireline networks, often the protocols are classified into layers.

- Layering is a form of hierarchical modularity.

- The higher layer uses the service provided by the lower layer. But it does not need to know the inner working of the lower layer

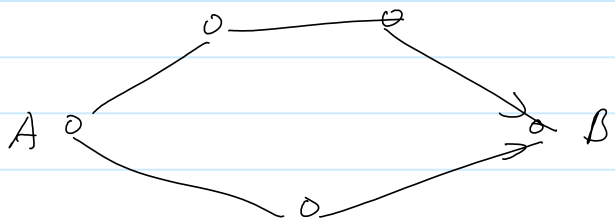


- Benefits of Modularity.
  - easy to understand
  - easy to change.

- However, for wireless networks, examples have been found where such a layering architecture can limit performance.

Example.

- Typically, routing is designed to minimize the # of hops



- Tend to use "long" links.

- In wireless networks, long transmission link can suffer from a low SINR

$\Rightarrow$ poor end-to-end performance.

- It would be better if the routing protocol takes into account the physical-layer characteristics.

- Pitfalls of Cross-layer Design

  - loss of modularity

  - fragile solution that is hard to change.

  - Would be highly desirable if we can still obtain "modular" solutions

— Need duality / decomposition.

Cross-layer formulation

Sunday, February 01, 2009 12:25 PM

In an optimization approach, it is not difficult to incorporate controls at multiple layers into a unified optimization problem.

- Physical layer:
  - power control, water-filling
  - uses rate-power function
- MAC:
  - scheduling
- Network Layer:
  - multi-path routing
  - node-balance equation
- Transport layer
  - utility maximization
  - revenue maximization

So we have various combinations.
Key consideration is
- convexity
- distributed/decomposed solution.

<u>One way of putting all together</u>

$$\max \sum_s U_s(x_s) \quad \text{- utility/congestion control}$$

$$\text{sub to} \quad x_s = \sum_j x_{sj} \quad \text{- routing/load-balancing.}$$

rate of user s on path j;

rate of user s on path j

$H_{sj}^l = 1$ if path j of user s uses link l

$$\sum_{sj} H_{sj}^l x_{sj} \le r_l$$

ρ_k: (given) prob. of channel state k

$$r_l \le \sum_{k=1}^{K} \rho_k r_l^k, \forall l$$

(with $\sum \rho_k = 1$)

- K channel states
- each may use a different decision (schedule k).

$$r_l^k \le g_l^k(\vec{p}^k)$$

- power control adaptive coding/modulation

rate-power function

power assignment for schedule k.

Other formulations

- replace H_{sj}^l by node-balance eqn
- use more than one schedules per channel-~~state~~
- The set of schedules can be very large.

Ref: Lin, Shroff & Srikant, "A Tutorial on Cross-Layer Optimization in Wireless Networks", IEEE Journal on Selected Areas in Comm, Special Issue on "Non-Linear Optimization of Comm Systems," 2006

- In general, not a convex problem.
  - The function $g_l^k(\cdot)$ may not be concave
  - For example, we may use Shannon's capacity

$$g_l^k(\vec{p}^k) = W \log \left(1 + \frac{p_l^k \cdot g_{ll}^k}{\sum_{h \neq l} p_h^k \cdot g_{hl}^k + N_l} \right)$$

  - However, the constraint $r_l^k \leq g_l^k(\vec{p}_k)$ is not convex in general!

- What can we do?
  - Convert to an equivalent convex problem
  - Find special cases that are convex
  - Approximate by a convex problem.