Lec1

Monday, January 12, 2009 9:19 PM

IERG 6120: Advanced Topics in IE I: Convex and Stochastic Optimization and Applications
Instructor: Xiaojun Lin
Tel: 3943-5587
E-mail: xjlin@ie.cuhk.edu.hk
Office hour: Friday 2:30-3:30pm or by appointment
Homepage: https://staff.ie.cuhk.edu.hk/~xjlin/IERG6120/

https://www.youtube.com/watch?v=vq5Rv4hB5jw

https://www.youtube.com/watch?v=wQylqaCl8Zo

Introduction

Thursday, January 08, 2009 10:10 AM

- In some research problems, you are given a system/protocol, and you wish to understand its performance.
- However, in many other problems, you may not be given an algorithm or protocol, but you are free to design your own.
- How can you design one that is in some sense the "best"?

---

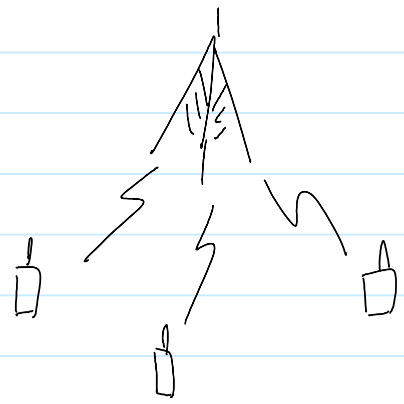
① Transmission on a wireless channel with fading:

Due to mobility & fading, the channel gain btw the basestation and the mobile changes with time.

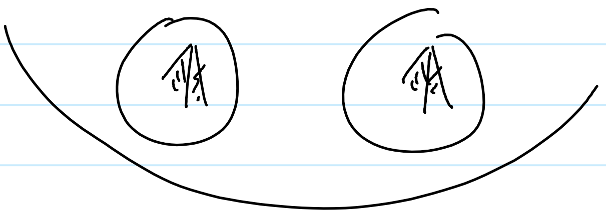
What should the transmission power at the BS be when the channel gain is g?

- Fixed power?
- Variable power to maintain fixed rate?
- What is the "best" power assignment?

What if the BS is transmitting to multiple users at the same time?



What if there are multiple BS or femto-cell?



15

Optimization

Monday, January 12, 2009 10:11 PM

In the first part of the class we will study convex optimizations, which will be a useful tool to answer such questions.

This process usually take the following steps:

① Built a system model

- $g(t)$: channel gain, randomly distributed with some distribution
- $r(t) = \log\left(1 + \frac{g(t) \cdot P(t)}{N}\right)$

② Identify the objectives to optimize

- maximize transmission rate, or
- minimize transmission power

③ Identify the constrains

- an upper bound on the transmission power
- a lower bound on the transmission rate

Without proper constraints, the optimization problem could become trivial.

(4) Formulate the optimization problem

Assume $g(t)$ is i.i.d, from a finite set of values $\{g_1, g_2, \cdots, g_M\}$.

Let $\quad q_k = \Pr\{g(0) = g_k\}, \; k=1,2,\cdots,M$

Let $\quad P_k$ denote the power assignment when the channel gain is g_k.

$$\max_{P_1, P_2, \cdots P_M} \quad \sum_{k=1}^{M} q_k \log\left(1+\frac{g_k \cdot P_k}{N}\right)$$

$$\text{subject to} \quad \sum_{k=1}^{M} q_k P_k \le P_{max}$$

(Q) Is this the only possible formulation?

(A) No. Can instead minimize power, delay, etc.

However, there is a reason why this is one of the more popular way of formulate the problem, as we will point out soon.

(5) Find the solution to this problem.

Such a solution may provide a direct relationship between $\{P_1, \cdots, P_M\}$ and the modelling parameters.

$$\vec{p}^* = f(\vec{g}, \vec{q})$$

At other times, a direct relationship may not be possible or convenient

- Not all equations have closed-form solutions

In that case, we may be happy with an iterative solution

$$\vec{p}(t+1) = f(\vec{p}(t) \mid \vec{g}, \vec{q})$$

$$\vec{p}(t) \rightarrow \vec{p}^*$$

Be it direct solutions or iterative solutions, for practical reasons we often

- prefer online adaptive solutions over offline solutions
    - estimating model parameter can be time-consuming
    - or inaccurate
    - or model parameter can change over time

- prefer distributed solutions over

centralized solutions

- single point of failure
- lack of scalability.

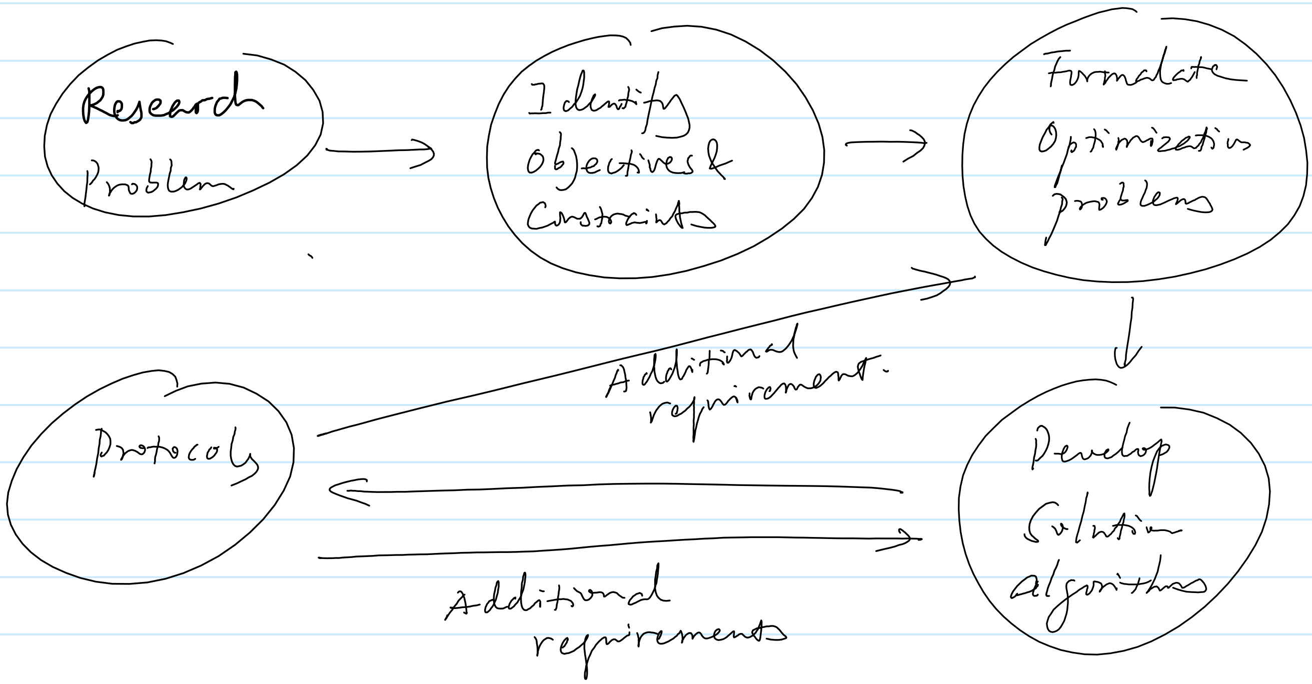
An online distributed solution can potentially be turned into a practical protocol that can be used in real systems.

25

In Summary

Monday, January 12, 2009 10:24 PM

In summary:



- distributed
- asynchronous
- imprecise or delayed feedback
- randomness
- need to look like an existing protocol

30

Convex problems

Thursday, January 08, 2009 11:16 AM

Unfortunately, not all problems formulated are that straightforward to come up with a solution.

Some problems are harder than others.

In this class, we will study the so-called convex problems

Some features

- the control variables are real numbers
  - not integers

- the objective function & constraints are continuous
  - not step-function (for example)

- the objective function & constraints are convex (very important).

Such problems are called convex optimization problems. We are interested in convex problems because

- They can model many (not all) interesting problems
- There are well-studied mechanisms to

develop solutions that are efficient & reliable.

Technical
<u>More features</u>

- local minimum is also the global minimum
- no duality gap: we can solve a convex problem through its dual
- dual program may lead to distributed solutions.

A lot of these convenience is due to convexity.

This class

Thursday, January 08, 2009 11:25 AM

However, it is not straight-forward to formulate a problem into a convex form

- More of an art then a science.
- There are tons of examples with success.

The goal of this class (first part).

- understand the characteristics of convex problems & common approaches to solve them
- gain some experience in formulating research problems into convex programs & solve them
- understand the issues & challenges to develop practical protocols based on convex optimization.

The class will cover:

- Fundamentals of theory
    - convexity
    - duality
    - convergence

- Applications in Communications & Networking
  - PHY/MAC: Power control, scheduling, random access
  - Transport: Congestion control, TCP
  - Routing: Multipath routing, network flow
  - Cross-layer design for wireless
- Applications in statistical learning
  - Regression
  - Classification.

We will use these applications to illustrate how convex optimization is used in practical problems.

50

Stochastic Optimization

Saturday, September 2, 2023 4:16 PM

- The second part of the class will study stochastic optimization
  - Specifically, we will study MDP (Markov Decision Program)

- Roughly speaking, stochastic optimization are needed when the objective/constraints are more "stringent"
  - long-term average rate vs. rate over the immediately-following short interval
  - long-term averge power constraint vs. finite power constraint from a small battery
  - Queue-length limit, delay limit, etc.

- As we will see later in the first part of the class, MDP can be seen as a special case of convex program
  - Key concepts from the first part of the class can still apply.

- However, MDP is sufficiently different so that solutions can be developed based on their special structure solutions can be developed based on their special structure.

- We will study the foundation theory of MDP & how to use it to solve research problems.