IERG6120: Homework 2

Due: Sept. 25, 2023 in class

Convex Functions

1. Let $f(x)$ be a function of $\mathbf{R} \to \mathbf{R}$ (i.e., it is a function of real numbers). Show that $f(x)$ is convex if and only if $\mathrm{dom} f$ is convex and $f(y) \geq f(x) + f'(x)(y-x)$ for all $x, y \in \mathrm{dom} f$. (Please follow and complete the proof steps described in class.)

2. Are these sets convex? (*Hint:* Write the set in a form $\{x | f(x) \leq 0\}$ where $f(x)$ is a convex function.)

   (a) Let λ be the arrival rate to an $M/M/1$ queueing system, and let d be a delay threshold. The set of λ such that the expected delay $E[D]$ is less than a given value d. What if d is also a variable, i.e., the set of (λ, d) such that the expected delay in the $M/M/1$ queueing system is less than d. (Note: The expected delay $E[D]$ of an $M/M/1$ queue with service rate μ and arrival rate λ is given by $E[D] = 1/(\mu - \lambda)$. For this problem, you can assume that μ is fixed.)

   (b) Let P_0 denote the transmission power of a given user. Let P_i, $i = 1, ..., I$ be the interference power of other users. Assume that the transmission rate of user 0 follows the Shannon capacity. The set of vectors $[P_0, P_1, ..., P_I]$ such that the transmission rate of user 0 is greater than a value r. What if r is also a variable and is a component of the vector to be considered. (Note: the Shannon capacity C of a channel with transmission power P, interference power I, and noise power N is given by $C = W \log(1 + P/(I+N))$, where W is the bandwidth of the system. For this problem, you can assume that W and N are fixed. The interference power I corresponds to the total transmission power of users other than user 0.)

   (c) Let x and y denote the length and width, respectively, of a rectangle contained in a circle with radius r. The set of such (x, y) for a given r, and the set of such (x, y, r).

3. Problem 3.5, 3.16 (part a/c/e), 3.21 (part a) and 3.22 (part a/c/e) of Boyd and Vandenberghe. (You don't need to worry about the quasi-convex and quasi-concave part of the problems.)