IERG6120: Homework 1

Due: Sept. 13, 2023 in class

Convex Sets

1. Define the affine hull of a set C as the smallest affine set containing C. You are asked to show that the affine hull of a set C is given by

$$\mathrm{aff}C = \{\theta_1 x_1 + \theta_2 x_2 + ... + \theta_k x_k | x_1, x_2, ..., x_k \in C, \theta_1 + \theta_2 + ... + \theta_k = 1\}.$$

Please prove it by following the two steps below: Let A denote the set on the right-hand-side. Note that A obvious contains C.

   (i) Show that A is an affine set.

   (ii) Suppose that there is another affine set B that contains C. Show that $A \subset B$.

2. Let $C \subset \mathrm{R}^3$ be the set that contains three points $(1, 0, 0), (0, 2, 0)$ and $(0, 0, 3)$. Write down the affine hull and convex hull of C. Please provide the description of these sets in two ways: one that uses θ's and one that does not use θ's (i.e., based on geometry).

3. Show that any intersection of convex sets is still a convex set.

4. Problem 2.11 and 2.12 (parts a/c/e/g) on page 61 of Boyd and Vandenberghe.

Convex Functions

1. A function $||x||$ for $x \in \mathbf{R}^n$ is said to be a *norm* if it satisfies the following properties

   (i) $||x|| \geq 0$ for all $x \in \mathbf{R}^n$.

   (ii) $||x|| = 0$ if and only if $x = 0$.

   (iii) $||\alpha x|| = |\alpha| \ ||x||$ for any real scalar α, where $|\alpha|$ denote the absolute value of α.

   (iv) (Triangle Inequality) $||x + y|| \leq ||x|| + ||y||$.

   Show that any norm is a convex function.