

# IERG6120: Homework 1

Due: Sept. 11, 2024 in class

## Convex Sets

1. Define the affine hull of a set  $C$  as the smallest affine set containing  $C$ . You are asked to show that the affine hull of a set  $C$  is given by

$$\text{aff}C = \{\theta_1x_1 + \theta_2x_2 + \dots + \theta_kx_k \mid x_1, x_2, \dots, x_k \in C, \theta_1 + \theta_2 + \dots + \theta_k = 1\}.$$

Please prove it by following the two steps below: Let  $A$  denote the set on the right-hand-side. Note that  $A$  obviously contains  $C$ .

- (i) Show that  $A$  is an affine set.
  - (ii) Suppose that there is another affine set  $B$  that contains  $C$ . Show that  $A \subset B$ .
2. Let  $C \subset \mathbb{R}^3$  be the set that contains three points  $(1, 0, 0)$ ,  $(0, 2, 0)$  and  $(0, 0, 3)$ . Write down the affine hull and convex hull of  $C$ . Please provide the description of these sets in two ways: one that uses  $\theta$ 's and one that does not use  $\theta$ 's (i.e., based on geometry).
  3. Show that any intersection of convex sets is still a convex set.
  4. Problem 2.11 and 2.12 (parts a/c/e/g) on page 61 of Boyd and Vandenberghe.