Reverse Signal-Aligned Network Coding in Interference Channels with Limited Transmitter Cooperation

Tse-Tin Chan and Tat-Ming Lok
Department of Information Engineering, The Chinese University of Hong Kong
Email: {ctt014, tmlok}@ie.cuhk.edu.hk

***Abstract*—In this paper, we study two-user time-varying interference channels with limited transmitter cooperation. The transmitters are connected to a central processor via wired cooperation links with individual limited capacities. The capacities of each cooperation link are just comparable to that of each link in the interference channel. We propose a reverse signal-aligned network coding (RSNC) scheme. We prove that our RSNC scheme achieves full degrees of freedom (DoF) by utilizing signal alignment and physical-layer network coding. The main idea of our RSNC scheme is that the transmitted signals are aligned at the receivers by signal alignment and the interfering signals are cancelled with each other by physical-layer network coding (PNC). This idea can be achieved by properly designing the network-coded messages conveyed from the central processor to the transmitters and the precoding matrices of the transmitters. Simulation results verify the performance of our RSNC scheme. The results also show that our scheme outperforms the orthogonal transmission scheme in the two-user case.**

***Index Terms*—degrees of freedom (DoF), distributed MIMO, interference alignment (IA), limited backhaul, physical-layer network coding (PNC)**

I. Introduction

Interference mitigation has become more important with the rapid growth of wireless devices. In this paper, we focus on interference channels with limited transmitter cooperation. We assume a central processor is connected to the transmitters via independent wired cooperation links with individual limited capacities. The central processor assigns the messages to the transmitters and then the transmitters convey the messages to the receivers. The capacity of a cooperation link is just greater than the rates of the links from the transmitter connected to that cooperation link to each receiver. The situation in which the transmitters are interconnected through cooperation links and one of the transmitters acts as the central processor is a special case of our channel model. This channel model has been widely investigated in many researches [1]–[6] such as those about cloud radio access network (C-RAN), coordinated multipoint (CoMP), distributed multiple-input multiple-output (MIMO) system, wireless local area network (WLAN), etc.

There are many schemes proposed to ease the interference problem in wireless communications. Interference alignment (IA) and physical-layer network coding (PNC) are promising examples. IA was initially developed in [7]–[9]. The main idea is aligning unwanted signals at each receiver so as to minimize the dimensions of interfering signal subspaces. Reference [10] extends the idea of IA to signal alignment in order to investigate the degrees of freedom (DoF) of the MIMO Y channel, in which three users exchanged independent messages with each other through an intermediate relay. PNC was proposed in [11] for two-way relay channels in which two users exchange packets with the aid of an intermediate relay. PNC demodulates superimposed signals into network-coded data by utilizing the additive property of electromagnetic (EM) waves. With proper use of the network-coded data, interference can be cancelled at the receivers in two-way relay channels. Some researches such as [10], [12]–[15] showed that employing signal alignment together with PNC is a promising way to mitigate the interference. Moreover, [14] proposed a scheme for distributed MIMO block-fading channels, in which the channel coefficients remained constant over a block of symbols. The performance of that scheme is limited by the numbers of antenna per node.

In this paper, we propose a reverse signal-aligned network coding (RSNC) scheme for two-user time-varying interference channels with limited transmitter cooperation. We consider the transmitters are connected to the central processor through wired cooperation links with individual limited capacities. Our RSNC scheme consists of three parts. First, the signals from the transmitters are aligned at each receiver by designing the precoding matrices of the transmitters over multiple symbol extensions of the time-varying channel. Second, the aligned signals are decoded at each receiver into noiseless linear combinations of transmitted messages, also know as network-coded messages. The decoding of linear equations of messages can be achieved by compute-and-forward or other PNC strategies. Third, the messages conveyed from the central processor to the transmitters are network-coded. These network-coded messages are designed so as to cancel all the unintended messages at each receiver by network coding.

If the cooperation links between the central processor and the transmitters have infinite capacity, the analysis is straightforward. However, the capacities of the cooperation links are limited in practice. In our system model, analog signals or raw signal samples cannot be transmitted from the central

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processor to the transmitters due to the limited capacities of the wired cooperation links. The capacity of each cooperation link is just sufficient to convey the digital messages or the linear combinations of them. The main contribution of this paper is the RSNC scheme for time-varying interference channels with limited transmitter cooperation. The DoF, also known as the multiplexing gain or the capacity pre-log, is useful in characterizing the capacity behavior in the high SNR regime. It provides a first-order approximation to the capacity. We show that our proposed RSNC scheme can achieve full DoF. In other words, each user can communicate as if there is no interference at high SNR.

Our RSNC scheme extends the SNC scheme [15] which is originally designed for interference channels with limited receiver cooperation. There is a significant difference between limited transmitter and receiver cooperations. In the case of receiver cooperation, the final destination of the messages is the central processor. The central processor can just collect linearly independent equations of the messages from the receivers and then recover the original messages of the transmitters. By contrast, in the case of transmitter cooperation, the final destination is the receivers. In other words, we need to mitigate all interference at each receiver. Hence, SNC scheme [15] for limited receiver cooperation cannot be applied directly into the channel model considered in this paper.

Notations: In this paper, we use letters of bold upper case, bold lower case, and lower case to denote matrices, vectors, and scalars respectively. The set of all complex-valued $m \times n$ matrices is represented by $\mathbb{C}^{m\times n}$. The set of all $m \times n$ matrices in a finite field of size q is expressed by $\mathbb{F}_q^{m\times n}$. The addition operation and the multiplication operation over that finite field are denoted by $\oplus$ and $\otimes$ respectively. $\mathbb{Z}^+$ means the set of all positive integers and $\mathbf{I}_d$ denotes the $d \times d$ identity matrix. Moreover, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $\|\cdot\|_{\mathrm{F}}$, and $\mathbb{E}[\cdot]$ denote conjugate transpose, transpose, Frobenius norm, and statistical expectation respectively.

II. System Model

We consider a time-varying interference channel with limited transmitter cooperation which consists of a central processor, 2 transmitters, and 2 receivers as shown in Fig. 1. Each transmitter is connected to the central processor via an independent noiseless cooperation link. The rate-constraint of a cooperation link is just greater than the rates of the links from the transmitter connected to that cooperation link to each receiver. Each transmitter and receiver is equipped with one antenna. Unique indices $k \in \{1, 2\}$ and $l \in \{1, 2\}$ are assigned to each transmitter and receiver respectively. The overall transmission consists of two phases. In the first phase, the central processor sends the processed messages, which can be coded symbols, or the linear equations of the messages in the same finite field to the transmitters through the cooperation links. In the second phase, the transmitters convey the messages given by the central processor to the receivers in the interference channel. We assume that the transmitters send signals synchronously and share the same communication resources such as time, frequency, and code. We also assume instantaneous channel state information (CSI) is globally available.

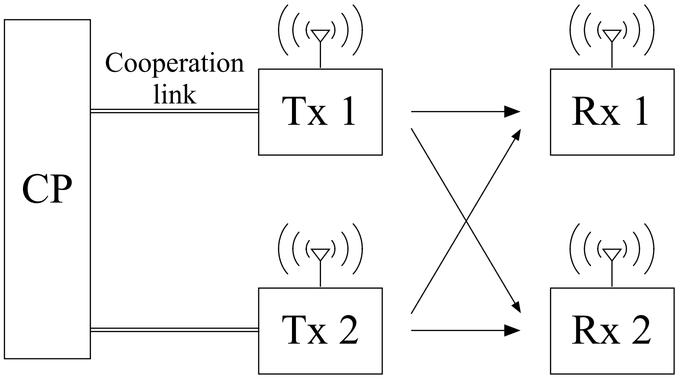

Fig. 1. A two-user time-varying interference channel with limited transmitter cooperation, which consists of a central processor (CP), two transmitters (Tx), and two receivers (Rx).

A. From the Central Processor to the Transmitters

The system adopts an $N = n + 1$ symbol extension of the time-varying channel where $n \in \mathbb{Z}^+$. The N symbol extension means that the N symbols transmitted from each transmitter over N slots are collectively denoted as a super-symbol. The central processor wants to convey message vector $\mathbf{b}_l(t) \in \mathbb{F}_q^{N\times 1}$ to receiver l with the aid of the transmitters where

$$\mathbf{b}_l(t) = \begin{bmatrix} b_l^{(1)}(t) & b_l^{(2)}(t) & \cdots & b_l^{(N)}(t) \end{bmatrix}^{\mathrm{T}}. \quad (1)$$

The i-th slot of the N symbol extension is indicated by the superscript (i) in this paper. Message vector $\mathbf{b}_l(t)$ is the N symbol extension of independent and identically distributed (i.i.d.) message $b_l(t)$. In this paper, the time index $t \in \mathbb{Z}^+$ can be used to denote time, frequency, or time-frequency slots.

In the first transmission phase, the central processor passes the message vectors to a processing function $\omega(\cdot)$ where

$$(\tilde{\mathbf{b}}_1(t), \tilde{\mathbf{b}}_2(t)) = \omega(\mathbf{b}_1(t), \mathbf{b}_2(t)). \quad (2)$$

Processed message vector $\tilde{\mathbf{b}}_k(t) \in \mathbb{F}_q^{N\times 1}$ is the N symbol extension of processed message $\tilde{b}_k(t)$ for transmitter k where

$$\tilde{\mathbf{b}}_k(t) = \begin{bmatrix} \tilde{b}_k^{(1)}(t) & \tilde{b}_k^{(2)}(t) & \cdots & \tilde{b}_k^{(N)}(t) \end{bmatrix}^{\mathrm{T}}. \quad (3)$$

The central processor then sends processed message vector $\tilde{\mathbf{b}}_k(t)$ to transmitter k via an independent noiseless cooperation link.

B. From the Transmitters to the Receivers

In the second transmission phase, transmitter k modulates processed message vector $\tilde{\mathbf{b}}_k(t)$ sent from the central processor to signal vector $\mathbf{x}_k(t) \in \mathbb{C}^{N\times 1}$ where

$$\mathbf{x}_k(t) = \begin{bmatrix} x_k^{(1)}(t) & x_k^{(2)}(t) & \cdots & x_k^{(N)}(t) \end{bmatrix}^{\mathrm{T}}. \quad (4)$$

Signal vector $\mathbf{x}_k(t)$ is the N symbol extension of signal $x_k(t)$ and $\mathbb{E}[\mathbf{x}_k(t)\mathbf{x}_k^{\mathrm{H}}(t)] = \mathbf{I}_N$. Transmitter k then sends signal vector $\mathbf{x}_k(t)$ with linear precoding matrix $\mathbf{V}_k(t) \in \mathbb{C}^{N\times N}$ where

$$\mathbf{V}_k(t) = \begin{bmatrix} \mathbf{v}_k^{(1)}(t) & \mathbf{v}_k^{(2)}(t) & \cdots & \mathbf{v}_k^{(N)}(t) \end{bmatrix}. \tag{5}$$

The i-th column vector of $\mathbf{V}_k(t)$, $\mathbf{v}_k^{(i)}(t) \in \mathbb{C}^{N\times 1}$, is the precoding vector for signal $x_k^{(i)}(t)$ presented in (4). Let $p^{\max}(t)$ and $p_k(t)$ be the maximum transmit power of the system and the actual transmit power of transmitter k respectively. The transmit power constraint of the system is

$$\sum_{k=1}^{2} p_k(t) = \sum_{k=1}^{2} \|\mathbf{V}_k(t)\|_{\mathrm{F}}^2 \in [0, p^{\max}(t)]. \tag{6}$$

Received signal vector $\mathbf{y}_l(t) \in \mathbb{C}^{N\times 1}$ is the N symbol extension of received signal $y_l(t)$ at receiver l where

$$\begin{aligned}\mathbf{y}_l(t) &= \begin{bmatrix} y_l^{(1)}(t) & y_l^{(2)}(t) & \cdots & y_l^{(N)}(t) \end{bmatrix}^{\mathrm{T}} \\ &= \sum_{k=1}^{2} \mathbf{H}_{l,k}(t)\mathbf{V}_k(t)\mathbf{x}_k(t) + \mathbf{n}_l(t).\end{aligned} \tag{7}$$

Diagonal channel matrix $\mathbf{H}_{l,k}(t) \in \mathbb{C}^{N\times N}$ is the N symbol extension of channel coefficient $h_{l,k}(t)$ where

$$\mathbf{H}_{l,k}(t) = \begin{bmatrix} h_{l,k}^{(1)}(t) & 0 & \dots & 0 \\ 0 & h_{l,k}^{(2)}(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_{l,k}^{(N)}(t) \end{bmatrix} \tag{8}$$

and $h_{l,k}(t)$ is the CSI of the link from transmitter k to receiver l. Furthermore, noise vector $\mathbf{n}_l(t) \in \mathbb{C}^{N\times 1}$ is the N symbol extension of noise term $n_l(t)$ with variance $\sigma_l^2(t)$ at receiver l where

$$\mathbf{n}_l(t) = \begin{bmatrix} n_l^{(1)}(t) & n_l^{(2)}(t) & \cdots & n_l^{(N)}(t) \end{bmatrix}^{\mathrm{T}}. \tag{9}$$

We assume all channel coefficients are i.i.d. zero-mean unit-variance complex Gaussian random variables. Hence, $\mathbf{H}_{l,k}$ has full rank $N = n + 1$ almost surely because the elements of $\mathbf{H}_{l,k}$ are drawn independently from a continuous distribution. We also assume all noise terms are i.i.d. complex additive white Gaussian noise (AWGN).

Receiver l decodes received signal vector $\mathbf{y}_l(t)$ by linear filtering matrix $\mathbf{U}_l(t) \in \mathbb{C}^{N\times N}$ where

$$\mathbf{U}_l(t) = \begin{bmatrix} \mathbf{u}_l^{(1)}(t) & \mathbf{u}_l^{(2)}(t) & \cdots & \mathbf{u}_l^{(N)}(t) \end{bmatrix}. \tag{10}$$

The filtered signal vector at receiver l is $\mathbf{x}'_l(t) \in \mathbb{C}^{N\times 1}$ that

$$\begin{aligned}\mathbf{x}'_l(t) &= \begin{bmatrix} x_l'^{(1)}(t) & x_l'^{(2)}(t) & \cdots & x_l'^{(N)}(t) \end{bmatrix}^{\mathrm{T}} \\ &= \mathbf{U}_l^{\mathrm{H}}(t)\mathbf{y}_l(t) \\ &= \sum_{k=1}^{2} \mathbf{U}_l^{\mathrm{H}}(t)\mathbf{H}_{l,k}(t)\mathbf{V}_k(t)\mathbf{x}_k(t) + \mathbf{U}_l^{\mathrm{H}}(t)\mathbf{n}_l(t).\end{aligned} \tag{11}$$

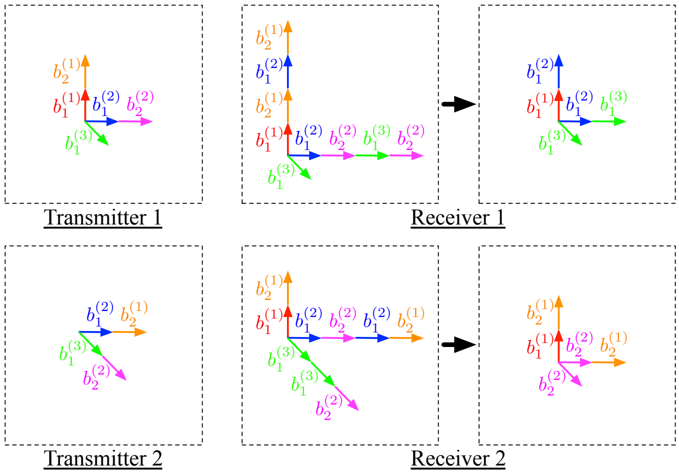

Fig. 2. RSNC scheme in the two-user interference channel with limited transmitter cooperation.

Receiver l then demodulates filtered signal vector $\mathbf{x}'_l(t)$ to demodulated message vector $\mathbf{b}'_l(t) \in \mathbb{F}_q^{N\times 1}$ where

$$\mathbf{b}'_l(t) = \begin{bmatrix} b_l'^{(1)}(t) & b_l'^{(2)}(t) & \cdots & b_l'^{(N)}(t) \end{bmatrix}^{\mathrm{T}}. \tag{12}$$

If $\mathbf{b}'_l(t) \neq \mathbf{b}_l(t)$ for any l, we say a decoding error occurs. For the sake of simplicity, the time index t is omitted in the rest of this paper.

C. Degrees of Freedom

The capacity of conveying messages to receiver l at SNR ρ can be expressed as

$$C_l(\rho) = d_l \log_2(\rho) + \mathrm{o}(\log_2(\rho)). \tag{13}$$

$\mathrm{o}(\log_2(\rho))$ is a function that $\frac{\mathrm{o}(\log_2(\rho))}{\log_2(\rho)}$ tends to zero when ρ tends to infinity. The DoF for conveying messages to receiver l, which is also known as the multiplexing gain, can be found by

$$d_l = \lim_{\rho\to\infty} \frac{C_l(\rho)}{\log_2(\rho)}. \tag{14}$$

III. Illustrative Example

We present a simple example of our RSNC scheme as illustrated in Fig. 2. The detailed proof and description are shown in Section IV. We show the users can achieve total $d_{\sum} = 5$ DoF over an $N = 3$ symbol extension in two-user time-varying interference channels with limited transmitter cooperation by our RSNC scheme.

The central processor wants to convey message vector $\begin{bmatrix} b_1^{(1)} & b_1^{(2)} & b_1^{(3)} \end{bmatrix}^{\mathrm{T}}$ to receiver 1 and message vector $\begin{bmatrix} b_2^{(1)} & b_2^{(2)} \end{bmatrix}^{\mathrm{T}}$ to receiver 2 with the aid of the transmitters. In order to achieve this goal, the central processor firstly sends processed message vectors

$$\begin{bmatrix} \tilde{b}_1^{(1)} & \tilde{b}_1^{(2)} & \tilde{b}_1^{(3)} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} b_1^{(1)} \oplus b_2^{(1)} & b_1^{(2)} \oplus b_2^{(2)} & b_1^{(3)} \end{bmatrix}^{\mathrm{T}} \tag{15}$$

to transmitter 1 and

$$\begin{bmatrix}\tilde{b}_2^{(1)} & \tilde{b}_2^{(2)}\end{bmatrix}^{\mathrm{T}} = \begin{bmatrix}b_1^{(2)} \oplus b_2^{(1)} & b_1^{(3)} \oplus b_2^{(2)}\end{bmatrix}^{\mathrm{T}} \tag{16}$$

to transmitter 2 in the first phase. Then transmitter 1 modulates processed message $\tilde{b}_1^{(i)}$ to signal $x_1^{(i)}$ where $i \in \{1,2,3\}$ while transmitter 2 modulates processed message $\tilde{b}_2^{(j)}$ to signal $x_2^{(j)}$ where $j \in \{1,2\}$.

Considering the communication in the second phase, i.e. the interference channel, the transmitters precodes the signals in order to align the transmitted signals in a way to follow our RSNC scheme. For the sake of simplicity, we leave the way which performs the signal alignment in our RSNC scheme to the next section. The transmitted signals are aligned and demodulated as follows:

- At receiver 1, signal $x_1^{(1)}$ is aligned with signal $x_2^{(1)}$ while signal $x_1^{(2)}$ is aligned with signal $x_2^{(2)}$. Receiver 1 demodulates the received signals and obtains

$$\begin{aligned}\begin{bmatrix}\tilde{b}_1^{(1)} \oplus \tilde{b}_2^{(1)} \\ \tilde{b}_1^{(2)} \oplus \tilde{b}_2^{(2)} \\ \tilde{b}_1^{(3)}\end{bmatrix} &= \begin{bmatrix}(b_1^{(1)} \oplus b_2^{(1)}) \oplus (b_1^{(2)} \oplus b_2^{(1)}) \\ (b_1^{(2)} \oplus b_2^{(2)}) \oplus (b_1^{(3)} \oplus b_2^{(2)}) \\ b_1^{(3)}\end{bmatrix} \\ &= \begin{bmatrix}b_1^{(1)} \oplus b_1^{(2)} \\ b_1^{(2)} \oplus b_1^{(3)} \\ b_1^{(3)}\end{bmatrix}.\end{aligned} \tag{17}$$

  Then receiver 1 can get message $b_1^{(3)}$ directly and message $b_1^{(2)}$ by performing network coding, i.e., $(b_1^{(2)} \oplus b_1^{(3)}) \oplus b_1^{(3)} = b_1^{(2)}$. Message $b_1^{(1)}$ can be obtained likewise.
- At receiver 2, signal $x_1^{(2)}$ is aligned with signal $x_2^{(1)}$ while signal $x_1^{(3)}$ is aligned with signal $x_2^{(2)}$. Receiver 2 demodulates the received signals and obtains

$$\begin{aligned}\begin{bmatrix}\tilde{b}_1^{(1)} \\ \tilde{b}_1^{(2)} \oplus \tilde{b}_2^{(1)} \\ \tilde{b}_1^{(3)} \oplus \tilde{b}_2^{(2)}\end{bmatrix} &= \begin{bmatrix}(b_1^{(1)} \oplus b_2^{(1)}) \\ (b_1^{(2)} \oplus b_2^{(2)}) \oplus (b_1^{(2)} \oplus b_2^{(1)}) \\ b_1^{(3)} \oplus (b_1^{(3)} \oplus b_2^{(2)})\end{bmatrix} \\ &= \begin{bmatrix}b_1^{(1)} \oplus b_2^{(1)} \\ b_2^{(2)} \oplus b_2^{(1)} \\ b_2^{(2)}\end{bmatrix}.\end{aligned} \tag{18}$$

  Finally, receiver 2 can obtain message $b_2^{(2)}$ directly and get message $b_2^{(1)}$ by performing network coding, i.e., $(b_2^{(2)} \oplus b_2^{(1)}) \oplus b_2^{(2)} = b_2^{(1)}$.

As a result, the users can achieve total $d_{\sum} = 5$ DoF over an $N = 3$ symbol extension in two-user time-varying interference channels with limited transmitter cooperation by our RSNC scheme.

IV. Reverse Signal-Aligned Network Coding

We describe the details of our RSNC scheme in this section. We show the users can achieve total $d_{\sum} = 2n+1$ DoF over an $N = n+1$ symbol extension in two-user time-varying interference channels with limited transmitter cooperation by our RSNC scheme.

First of all, we look at the second transmission phase which is the communication between the transmitters and the receivers in the interference channel. Transmitter 1 modulates $(n+1) \times 1$ processed message vector $\tilde{\mathbf{b}}_1$, which is sent from the central processor, to $(n+1) \times 1$ signal vector $\mathbf{x}_1$ while transmitter 2 modulates $n \times 1$ processed message vector $\tilde{\mathbf{b}}_2$ to $n \times 1$ signal vector $\mathbf{x}_2$. Afterward transmitter 1 sends signal vector $\mathbf{x}_1$ with $(n+1) \times (n+1)$ linear precoding matrix $\mathbf{V}_1$ while transmitter 2 sends signal vector $\mathbf{x}_2$ with $(n+1) \times n$ linear precoding matrix $\mathbf{V}_2$.

We set up the following signal alignment constraints for the transmitters:

$$\mathbf{H}_{1,2}\mathbf{V}_2 \prec \mathbf{H}_{1,1}\mathbf{V}_1, \tag{19}$$

$$\mathbf{H}_{2,2}\mathbf{V}_2 \prec \mathbf{H}_{2,1}\mathbf{V}_1 \tag{20}$$

where $\mathbf{Q} \prec \mathbf{P}$ denotes that the column vectors of matrix $\mathbf{Q}$ is a subset of those of matrix $\mathbf{P}$ in this paper. In our RSNC scheme, the signals are just required to be aligned in the same direction. For example, considering alignment constraint (19), the alignment constraint can be $\mathbf{H}_{1,2}\mathbf{V}_2 \prec \alpha\mathbf{H}_{1,1}\mathbf{V}_1$ where α is a scalar. We do not consider the optimization in this aspect because we focus on introducing our RSNC scheme in this paper.

In order to fulfill alignment constraints (19) and (20) and achieve the ideas of our RSNC scheme, we can set the linear precoding matrices of the transmitters as follows:

$$\mathbf{V}_1 = \left[\mathbf{G}_{1,2}^{n}\mathbf{w}, \mathbf{G}_{1,2}^{n-1}\mathbf{G}_{2,2}\mathbf{w}, \ldots, \mathbf{G}_{1,2}\mathbf{G}_{2,2}^{n-1}\mathbf{w}, \mathbf{G}_{2,2}^{n}\mathbf{w}\right], \tag{21}$$

$$\mathbf{V}_2 = \left[\mathbf{G}_{1,2}^{n-1}\mathbf{w}, \mathbf{G}_{1,2}^{n-2}\mathbf{G}_{2,2}\mathbf{w}, \ldots, \mathbf{G}_{1,2}\mathbf{G}_{2,2}^{n-2}\mathbf{w}, \mathbf{G}_{2,2}^{n-1}\mathbf{w}\right] \tag{22}$$

where $\mathbf{G}_{1,2} = \mathbf{H}_{1,1}^{-1}\mathbf{H}_{1,2}$, $\mathbf{G}_{2,2} = \mathbf{H}_{2,1}^{-1}\mathbf{H}_{2,2}$, and $\mathbf{w}$ is an arbitrary $(n+1) \times 1$ column vector. The column vectors in (21) and (22) are separated by commas due to space limitation. Here $\mathbf{V}_1$ is an $(n+1) \times (n+1)$ matrix and $\mathbf{V}_2$ is an $(n+1) \times n$ matrix. As mentioned above, we do not consider the optimization of the precoding vectors of the transmitters, hence vector $\mathbf{w}$ can be chosen arbitrarily. Without loss of generality, we assume

$$\mathbf{w} = \begin{bmatrix}1 & 1 & \cdots & 1\end{bmatrix}^{\mathrm{T}}. \tag{23}$$

Now we show that the signals from the transmitters are aligned at receiver 1. As the multiplications of diagonal matrices are commutative, the multiplications of the channel matrices and the linear precoding matrices at receiver 1 are

$$\begin{aligned}&\mathbf{H}_{1,1}\mathbf{V}_1 \\ &= [\mathbf{H}_{1,2}\mathbf{G}_{1,2}^{n-1}\mathbf{w}, \ldots, \mathbf{H}_{1,2}\mathbf{G}_{2,2}^{n-1}\mathbf{w}, \mathbf{H}_{1,1}\mathbf{G}_{2,2}^{n}\mathbf{w}],\end{aligned} \tag{24}$$

$$\begin{aligned}&\mathbf{H}_{1,2}\mathbf{V}_2 \\ &= [\mathbf{H}_{1,2}\mathbf{G}_{1,2}^{n-1}\mathbf{w}, \mathbf{H}_{1,2}\mathbf{G}_{1,2}^{n-2}\mathbf{G}_{2,2}\mathbf{w}, \ldots, \mathbf{H}_{1,2}\mathbf{G}_{2,2}^{n-1}\mathbf{w}],\end{aligned} \tag{25}$$

where $\mathbf{H}_{1,1}\mathbf{V}_1$ is an $(n+1) \times (n+1)$ matrix and $\mathbf{H}_{1,2}\mathbf{V}_2$ is an $(n+1) \times n$ matrix. The column vectors in (24) and (25) are separated by commas. The first n column vectors of $\mathbf{H}_{1,1}\mathbf{V}_1$ are the same as the column vectors of $\mathbf{H}_{1,2}\mathbf{V}_2$, therefore signal alignment constraint (19) is satisfied.

Receiver 1 decodes $(n+1)\times 1$ received signal vector $\mathbf{y}_1$ through $(n+1)\times(n+1)$ linear filtering matrix $\mathbf{U}_1^{\mathrm{H}} = (\mathbf{H}_{1,1}\mathbf{V}_1)^{-1}$. The filtered signal vector $\mathbf{x}_1'$ is

$$\begin{aligned}\mathbf{x}_1' &= \mathbf{U}_1^{\mathrm{H}}\mathbf{y}_1 \\ &= \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,1}\mathbf{V}_1\mathbf{x}_1 + \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,2}\mathbf{V}_2\mathbf{x}_2 + \mathbf{U}_1^{\mathrm{H}}\mathbf{n}_1. \end{aligned} \tag{26}$$

We can express filtered signal vector $\mathbf{x}_1'$ as

$$\begin{aligned}\mathbf{x}_1' &= \begin{bmatrix}\mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,1}\mathbf{V}_1 & \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,2}\mathbf{V}_2\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} + \mathbf{U}_1^{\mathrm{H}}\mathbf{n}_1 \\ &= \mathbf{F}_1\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} + \mathbf{U}_1^{\mathrm{H}}\mathbf{n}_1. \end{aligned} \tag{27}$$

Here $(n+1)\times(2n+1)$ alignment matrix $\mathbf{F}_1 = \begin{bmatrix}\mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,1}\mathbf{V}_1 & \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,2}\mathbf{V}_2\end{bmatrix}$ and it can also be regarded as the effective channel matrix at receiver 1. In PNC demodulation, we treat the aligned signal (e.g. $x_1^{(1)} + x_2^{(1)}$) as an unknown for demodulation rather than demodulate the original signals (e.g. $x_1^{(1)}$ and $x_2^{(1)}$) individually. Receiver 1 demodulates filtered signal vector $\mathbf{x}_1'$ to $(n+1)\times 1$ network-coded message vector $\mathbf{b}_1'$ over GF(q) where

$$\begin{aligned}\mathbf{b}_1' &= \begin{bmatrix}\mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,1}\mathbf{V}_1 & \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,2}\mathbf{V}_2\end{bmatrix} \otimes \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix} \\ &= \mathbf{F}_1 \otimes \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix}. \end{aligned} \tag{28}$$

Afterward we look at the signals filtered and demodulated at receiver 2. The messages decoded at receiver 2 can be understood likewise. As the multiplications of diagonal matrices are commutative, the multiplications of the channel matrices and the linear precoding matrices at receiver 2 are

$$\begin{aligned}&\mathbf{H}_{2,1}\mathbf{V}_1 \\ &= [\mathbf{H}_{2,1}\mathbf{G}_{1,2}^{n}\mathbf{w}, \mathbf{H}_{2,2}\mathbf{G}_{1,2}^{n-1}\mathbf{w}, \ldots, \mathbf{H}_{2,2}\mathbf{G}_{2,2}^{n-1}\mathbf{w}], \end{aligned} \tag{29}$$

$$\begin{aligned}&\mathbf{H}_{2,2}\mathbf{V}_2 \\ &= [\mathbf{H}_{2,2}\mathbf{G}_{1,2}^{n-1}\mathbf{w}, \ldots, \mathbf{H}_{2,2}\mathbf{G}_{1,2}\mathbf{G}_{2,2}^{n-2}\mathbf{w}, \mathbf{H}_{2,2}\mathbf{G}_{2,2}^{n-1}\mathbf{w}], \end{aligned} \tag{30}$$

where $\mathbf{H}_{2,1}\mathbf{V}_1$ is an $(n+1)\times(n+1)$ matrix and $\mathbf{H}_{2,2}\mathbf{V}_2$ is an $(n+1)\times n$ matrix. The column vectors in (29) and (30) are separated by commas. The last n column vectors of $\mathbf{H}_{2,1}\mathbf{V}_1$ are the same as the column vectors of $\mathbf{H}_{2,2}\mathbf{V}_2$, therefore signal alignment constraint (20) is satisfied.

Receiver 2 applies $(n+1)\times(n+1)$ linear filtering matrix $\mathbf{U}_2^{\mathrm{H}} = (\mathbf{H}_{2,1}\mathbf{V}_1)^{-1}$ to decode $(n+1)\times 1$ received signal vector $\mathbf{y}_2$. The $(n+1)\times 1$ filtered signal vector $\mathbf{x}_2'$ is

$$\begin{aligned}\mathbf{x}_2' &= \mathbf{U}_2^{\mathrm{H}}\mathbf{y}_2 \\ &= \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,1}\mathbf{V}_1\mathbf{x}_1 + \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,2}\mathbf{V}_2\mathbf{x}_2 + \mathbf{U}_2^{\mathrm{H}}\mathbf{n}_2 \\ &= \begin{bmatrix}\mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,1}\mathbf{V}_1 & \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,2}\mathbf{V}_2\end{bmatrix}\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} + \mathbf{U}_2^{\mathrm{H}}\mathbf{n}_2 \\ &= \mathbf{F}_2\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\end{bmatrix} + \mathbf{U}_2^{\mathrm{H}}\mathbf{n}_2. \end{aligned} \tag{31}$$

Here $(n+1)\times(2n+1)$ alignment matrix $\mathbf{F}_2 = \begin{bmatrix}\mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,1}\mathbf{V}_1 & \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,2}\mathbf{V}_2\end{bmatrix}$, which can also be regarded as the effective channel matrix, affects the alignment of the received signals at receiver 2. Filtered signal vector $\mathbf{x}_2'$ is then demodulated to $(n+1)\times 1$ network-coded message vector $\mathbf{b}_2'$ over GF(q) where

$$\begin{aligned}\mathbf{b}_2' &= \begin{bmatrix}\mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,1}\mathbf{V}_1 & \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,2}\mathbf{V}_2\end{bmatrix} \otimes \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix} \\ &= \mathbf{F}_2 \otimes \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix}. \end{aligned} \tag{32}$$

Now, we focus on the first transmission phase. We show that an appropriate processing of the messages at the central processor can let the receivers decode their intended messages without interference. We combine the $(n+1)\times(2n+1)$ alignment matrices of receivers 1 and 2 and form a $(2n+2)\times(2n+1)$ alignment matrix of the system over GF(q) where

$$\mathbf{F} = \begin{bmatrix}\mathbf{F}_1\\ \mathbf{F}_2\end{bmatrix} = \begin{bmatrix}\mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,1}\mathbf{V}_1 & \mathbf{U}_1^{\mathrm{H}}\mathbf{H}_{1,2}\mathbf{V}_2\\ \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,1}\mathbf{V}_1 & \mathbf{U}_2^{\mathrm{H}}\mathbf{H}_{2,2}\mathbf{V}_2\end{bmatrix}. \tag{33}$$

Here the alignment matrix of the system, $\mathbf{F}$, has full rank $2n+1$ for some finite field sizes q. The proof is presented in [15, Lemma 2].

As alignment matrix of the system $\mathbf{F}$ has full rank, we can select any $2n+1$ linearly independent rows of $\mathbf{F}$ to form a $(2n+1)\times(2n+1)$ invertible matrix $\mathbf{F}'$. The receivers can obtain their intended messages without interference if the message vectors at the central processor are left multiplied by the inverse of $\mathbf{F}'$. In other words, the processed message vectors of the central processor are

$$\begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix} = \mathbf{F}'^{-1} \otimes \begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{bmatrix}. \tag{34}$$

The central processor conveys processed message vectors $\tilde{\mathbf{b}}_1$ and $\tilde{\mathbf{b}}_2$ to transmitters 1 and 2, respectively, via independent noiseless cooperation links. The details of the transmissions from the transmitters to the receivers are described at the beginning of this section. Here we first look at the received messages at receiver 1. We combine (28) and (34) and get

$$\mathbf{b}_1' = \mathbf{F}_1 \otimes \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix} = \mathbf{F}_1 \otimes \mathbf{F}'^{-1} \otimes \begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{bmatrix}. \tag{35}$$

The messages recovered at receiver 2 can be understood likewise. Combining (32) and (34), receiver 2 gets

$$\mathbf{b}_2' = \mathbf{F}_2 \otimes \begin{bmatrix}\tilde{\mathbf{b}}_1\\ \tilde{\mathbf{b}}_2\end{bmatrix} = \mathbf{F}_2 \otimes \mathbf{F}'^{-1} \otimes \begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{bmatrix}. \tag{36}$$

Owing to the relationships among $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}'^{-1}$, all receivers can obtain their intended messages without interference.

As a result, the system achieves total $2n+1$ DoF over an $N = n+1$ symbol extension for any positive integer n by our RSNC scheme. The sum DoF achieved by the RSNC scheme for the two-user time-varying interference channel with limited transmitter cooperation tends to two with a large value of n. In other words, full DoF can be achieved by our RSNC scheme. Notice that the achievement of full DoF is not limited by the number of antennas per node.

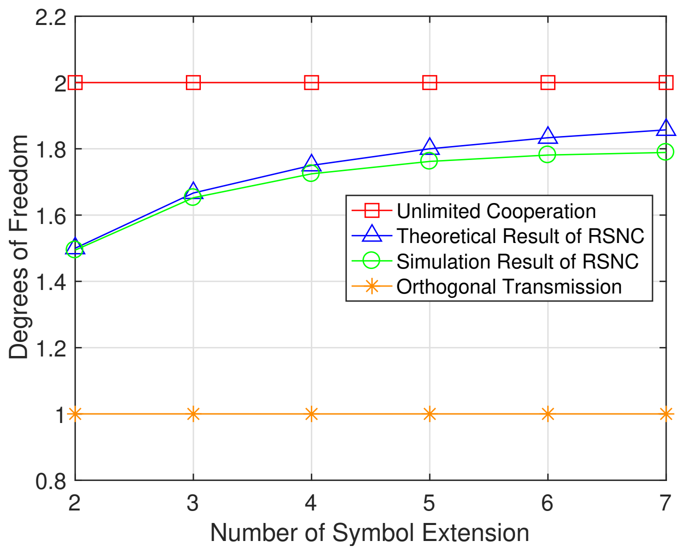

Fig. 3. Average degrees of freedom achieved in two-user time-varying interference channels with limited transmitter cooperation.

V. Simulation Results

In this section, we present the simulation results of our RSNC scheme in two-user time-varying interference channels with limited transmitter cooperation. We compare the DOF of our RSNC scheme with the unlimited cooperation scheme and the orthogonal transmission scheme. The unlimited cooperation scheme acts as an upper bound for comparison. We assume the cooperation links from the central processor to the transmitters have infinite capacity. The transmitters jointly precode the signals in order to nullify the interference at each receiver. In the orthogonal transmission scheme, the transmitters take turns sending signals to the intended receiver in order to prevent interfering with each other.

We assume the total transmit powers for all systems are the same and the noise variances at each node are the same. Simulation results are illustrated with respect to the number of symbol extension. We compute the DoF by dividing the sum-rate by $\log_2(\mathrm{SNR})$ at a very high SNR. The average DoF are computed using 1000 random channel realizations. The obtained results are shown in Fig. 3.

Fig. 3 verifies that the simulation results of our RSNC scheme coincide with its theoretical results. Moreover, Fig. 3 reveals that our RSNC scheme outperforms the orthogonal transmission scheme in the two-user case. This DoF performance improvement is achieved by properly aligning the signals by signal alignment so as to cancel the interference at the receivers by PNC.

VI. Conclusion

In this paper, we propose a RSNC scheme in two-user time-varying interference channels with limited transmitter cooperation. This channel model widely characterizes the scenarios in C-RAN, CoMP, distributed MIMO, WLAN, etc. In short, our RSNC scheme is to construct an appropriate alignment matrix of the system and then left multiply the message vectors at the central processor by the inverse of the matrix which consists of the linearly independent rows of that alignment matrix. We prove that our RSNC scheme is able to achieve arbitrarily close to full DoF. Simulation results verify the DoF performance of our scheme. Simulation results also show that our RSNC scheme achieves superior performance compared to the orthogonal transmission scheme in the two-user cases. The DoF improvement of our scheme mainly comes from properly aligning the signals by signal alignment for cancelling the interfering signals at the receivers by PNC demodulation. Work is currently underway to develop the general RSNC scheme for multi-user MIMO interference channels with limited transmitter cooperation.

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