Auction-Based Resource Allocation for Cooperative Communications

Jianwei Huang, Zhu Han, Mung Chiang, and H. Vincent Poor

Abstract—Distributed and efficient resource allocation is critical for fully realizing the benefits of cooperative communications in large scale communication networks. This paper proposes two auction mechanisms, the SNR auction and the power auction, that determine relay selection and relay power allocation in a distributed fashion. A single-relay network is considered first, and the existence and uniqueness of the Nash Equilibrium (i.e., the auction's outcome) are proved. It is shown that the power auction achieves the efficient allocation by maximizing the total rate increase, and the SNR auction is flexible in trading off fairness and efficiency. For both auctions, the distributed best response bid updates globally converge to the unique Nash Equilibrium in a completely asynchronous manner. The analysis is then generalized to networks with multiple relays, and the existence of the Nash Equilibrium is shown under appropriate conditions. Simulation results verify the effectiveness and robustness of the proposed algorithms.

Index Terms—Cooperative communications, game theory, auction theory, resource allocation, wireless networks

I. INTRODUCTION

COOPERATIVE communications take advantage of the broadcast nature of wireless channels, uses relay nodes as virtual antennas, and thus realizes the benefits of multiple-input-multiple-output communications in situations where physical multiple antennas are difficult to deploy (e.g., small sensor nodes). Various cooperative protocols have been proposed (e.g., [1]–[5]), such as amplify-and-forward, decode-and-forward, and estimate-and-forward. Although the physical layer performance of cooperative communications has been extensively studied in the context of small networks from an information theoretic point of view, there are still many open problems in terms of how to realize its full benefits in large-scale networks through efficient resource allocation. In particular, such performance optimization requires the knowledge of global channel information (including that for source-destination, source-relay, and relay-destination channels) and heterogeneous resource constraints among users. Centralized information exchange and coordination, however, typically do not scale well with the network size This motivates our study of distributed resource allocation algorithms for cooperative communications.

Resource allocation in cooperative networks has attracted attention from the research community only recently. Related work in this area can be divided into two categories: centralized (e.g., [6], [7]) and distributed (e.g., [8]–[12]). For example, in [6], Nosratinia and Hunter proposed a centralized approach for grouping and partner selection to achieve maximal network-wide diversity. In [7], Ng and Yu designed centralized resource allocation algorithm for power control, bandwidth allocation, relay selection and relay strategy choice in an Orthogonal Frequency-Division Multiple Access (OFDMA) based relay network. Bletsas *et al.* in [8] proposed a distributed relay selection scheme, where one user chooses the "best" end-to-end path among many relays based on instantaneous channel measurements. In [9], Savazzi and Spagnolini considered distributed power control for a single user multiple hop transmission. Himsoon *et al.* in [10] investigated the relay selection and power management schemes for lifetime extension in wireless sensor networks. Annavajjala *et al.* in [11] studied the optimal power control problem for different cooperative protocols under high Signal to Interference-plus-noise Ratio (SINR) approximation for a single user and multiple relays. Wang *et al.* in [12] investigated resource allocation for cooperative transmission using Stackelberg games. However, the above work did not consider distributed resource allocation for multiple users (source-destination pairs) and multiple relays, where each user is allowed to use more than one than one relay and each relay can help more than one users.

In this paper, we focus on answering the following two questions: 1) "When to relay", i.e., when is it beneficial for a user to use the relay(s)? and 2) "How to relay", i.e., how should a user choose the relay(s) and how should each relay allocate its resource (transmission power) among multiple competing users? We address these two issues by designing an auction-based resource allocation framework. Auction theory [13] has recently been introduced to several areas of wireless communications (e.g., time slot allocation [14], power control [15], and cognitive radio networking [16]). To the best of our knowledge, this paper is the first work that applies auction theory in designing resource allocation schemes for cooperative communications.

We consider two network objectives: *fairness* and *efficiency*. Both are difficult to achieve even in a centralized fashion. This is because users' rate increases are non-smooth and

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non-concave in the relay's transmission power, and thus the performance optimization problem is non-convex. We propose two auction mechanism, the SNR auction and the power auction, which achieve the desired network objectives in a distributed fashion under certain technical conditions. In both auctions, each user decides "when to relay" based on a simple threshold policy that is locally computable. The question of "how to relay" is answered by a simple weighted proportional allocation among users who use the relay. The power auction achieves the efficiency allocation, and the SNR auction offers a flexible approach to achieve various tradeoffs between fairness and efficiency. Moreover, we show that the desirable outcomes of the auctions (i.e., Nash Equilibrium) can be achieved by users' *greedy* best response updates with *local* information in a completely *asynchronous* manner. The results are quite general and are applicable to wireless networks with multiple relays at different locations with different resource constraints. Simulation results demonstrate optimality, fairness, and convergence of the proposed auctions.

Next we present the system model and network objectives in Section II. In Section III, two auction mechanisms are proposed, their mathematical properties are analyzed, and the algorithms for achieving the Nash Equilibrium in a distributed fashion are shown. Extensions to the multiple relay case are discussed in Section IV. Simulation results are shown in Section V and we conclude in Section VI.

II. SYSTEM MODEL AND NETWORK OBJECTIVES

A. System Model

We focus our discussions on the amplify-and-forward (AF) cooperation protocol [2] in this paper. Other cooperation protocols can be analyzed in a similar fashion. We begin by considering a simple system model as in Fig. 1, where there are one relay node r and a set $\mathcal{I} = (1, ..., I)$ of source-destination pairs (i.e., users). Each user i includes a source node s_i and a destination node d_i. The case of multiple relays will be discussed in Section IV.

We assume that different users transmit their signals using non-overlapping frequency bands and thus they do not interfere with each other. For each user i, the cooperative transmission consists of two phases.

In Phase 1, source s_i broadcasts its information so that it can be received by both destination d_i and relay r, respectively, as

$$Y_{s_i,d_i} = \sqrt{P_{s_i} G_{s_i,d_i}} X_{s_i} + n_{d_i}, \tag{1}$$

and

$$Y_{s_i,r} = \sqrt{P_{s_i} G_{s_i,r}} X_{s_i} + n_r. \tag{2}$$

Here P_{s_i} represents the fixed transmit power of source s_i, X_{s_i} is the transmitted information symbol with unit energy at source s_i, G_{s_i,d_i} and $G_{s_i,r}$ are the channel gains from s_i to destination d_i and relay r, respectively, and n_{d_i} and n_r are additive white Gaussian noises. Without loss of generality, we assume that the noise level is the same for all links and is denoted by σ^2. We also assume that the transmission frame length is small compared with the channel coherence time such that all channel gains are fixed during the time of interest. The
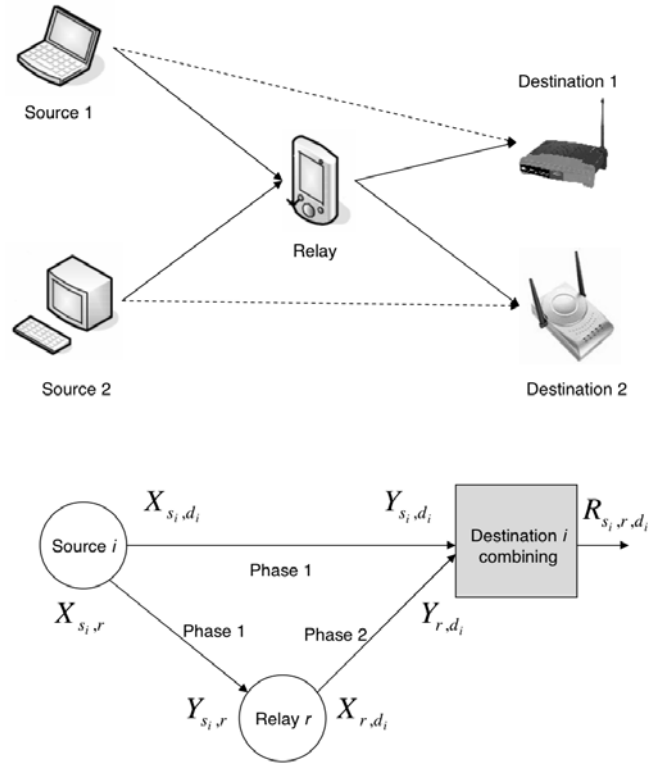

Fig. 1. System model for cooperative transmission (single relay case)

signal-to-noise ratio (SNR) obtained at destination d_i in Phase 1 is

$$\Gamma_{s_i,d_i} = \frac{P_{s_i} G_{s_i,d_i}}{\sigma^2}. \tag{3}$$

In Phase 2, relay r amplifies $Y_{s_i,r}$ with power P_{r,d_i} and forwards it to destination d_i. The received signal at destination d_i in Phase 2 is

$$Y_{r,d_i} = \sqrt{P_{r,d_i} G_{r,d_i}} X_{r,d_i} + n'_{d_i}, \tag{4}$$

where

$$X_{r,d_i} = \frac{Y_{s_i,r}}{|Y_{s_i,r}|} \tag{5}$$

is the unit-energy transmitted signal that relay r receives from source s_i in Phase 1, G_{r,d_i} is the channel gain from relay r to destination d_i, and n'_{d_i} is the received noise at Phase 2. Substituting (2) into (5), we can rewrite the received signal (4) as

$$Y_{r,d_i} = \frac{\sqrt{P_{r,d_i} G_{r,d_i}}(\sqrt{P_{s_i} G_{s_i,r}} X_{s_i,d_i} + n_r)}{\sqrt{P_{s_i} G_{s_i,r} + \sigma^2}} + n'_{d_i}. \tag{6}$$

Using (6), the relayed SNR at destination d_i in Phase 2 is

$$\Gamma_{s_i,r,d_i} = \frac{P_{r,d_i} P_{s_i} G_{r,d_i} G_{s_i,r}}{\sigma^2(P_{r,d_i} G_{r,d_i} + P_{s_i} G_{s_i,r} + \sigma^2)}. \tag{7}$$

Now we are ready to calculate the achievable rate at destination d_i. At a given transmission time, user i has two choices:

1) Use Phase 1 only and achieve rate (in bps/Hz)

$$R_{s_i,d_i} = W \log_2 (1 + \Gamma_{s_i,d_i}), \tag{8}$$
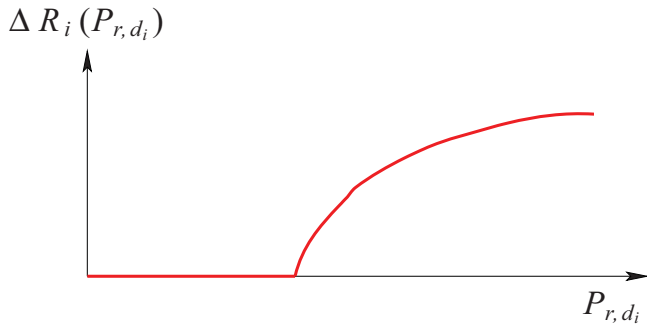
where W is the signal bandwidth.

Fig. 2.   Rate increase as a function of relay transmission power

2) Use both phases and achieve rate (at the output of maximal ratio combining)

$$R_{s_i,r,d_i} = \frac{1}{2}W \log_2 \left(1 + \Gamma_{s_i,d_i} + \Gamma_{s_i,r,d_i}\right). \quad (9)$$

The coefficient $1/2$ is due to the fact that cooperative transmission uses half of the resources (e.g., time slots, frequency bands, orthogonal codes). Since Γ_{s_i,r,d_i} is the extra SNR increase compared with the direct transmission, we also denote

$$\triangle \text{SNR}_i \triangleq \Gamma_{s_i,r,d_i}. \quad (10)$$

Comparing these two choices, the rate increase that user i can obtain comparing with the case of no relay is

$$\triangle R_i = \max \left\{R_{s_i,r,d_i} - R_{s_i,d_i}, 0\right\}, \quad (11)$$

which is nonnegative since the source can always choose not to use the relay and thereby obtain zero rate increase. $\triangle R_i$ is a function of the channel gains of the source-destination, source-relay and relay-destination links, as well as the transmission power of the source and the relay. In particular, $\triangle R_i$ is a non-decreasing, non-smooth, and non-concave function of the relay transmission power P_{r,d_i}, as illustrated in Fig. 2.

We assume that the source transmission power P_{s_i} (for all users i) and the relay's total power P are fixed. The relay determines the allocation of its transmission power among users, $\boldsymbol{P}_r \triangleq (P_{r,d_i}, \forall i \in \mathcal{I})$, such that the total power constraint is not violated, i.e.,

$$\boldsymbol{P}_r \in \mathcal{P}_r \triangleq \left\{\boldsymbol{P}_r \,\middle|\, \sum_{i \in \mathcal{I}} P_{r,d_i} \leq P, P_{r,d_i} \geq 0, \forall i \in \mathcal{I}\right\}. \quad (12)$$

B. Network Objectives: Efficiency and Fairness

We consider two different network objectives: *efficiency* and *fairness*. An efficient power allocation $\boldsymbol{P}_r^{\text{efficient}}$ maximizes the total rate increase of all users, i.e. solves the following problem,

$$\max_{\boldsymbol{P}_r \in \mathcal{P}_r} \sum_{i \in \mathcal{I}} \triangle R_i (P_{r,d_i}). \quad (13)$$

In many cases, an efficient allocation discriminates against users who are far away from the relay. To avoid this, we also consider a fair power allocation $\boldsymbol{P}_r^{\text{fair}}$, which solves the

following problem,

$$\min_{\boldsymbol{P}_r \in \mathcal{P}_r} c \quad (14)$$

$$\text{subject to } \frac{\triangle R_i (\triangle \text{SNR}_i)}{\partial (\triangle \text{SNR}_i)} = cq_i \cdot \mathbf{1}_{\{\triangle \text{SNR}_i > 0\}}, \forall i \in \mathcal{I}.$$

Here $\mathbf{1}_{\{\cdot\}}$ is the indicator function, and q_i is a user dependent priority parameter. When $q_i = 1$ for each i, all users who use the relay will have the same marginal utility, which leads to strict fairness among users.[1] It is possible to assign different weights to different users to achieve different Qualities of Service. One such example is to let

$$q_i = \frac{W}{2 \ln 2 \left(1 + \Gamma_{s_i,d_i}\right)} = \frac{\partial \triangle R_i (\triangle \text{SNR}_i)}{\partial (\triangle \text{SNR}_i)}\bigg|_{\triangle \text{SNR}_i = 0}, \quad (15)$$

i.e., q_i represents user i's eagerness to gain extra SNR increase by using the relay. The intuition behind Problem (14) is that for all users that choose to use the relay, the corresponding $\triangle \text{SNR}$ should be maximized subject to the "weighted marginal utility equalization" condition. This can be translated into the minimization of the common coefficient c due to the concavity of $\triangle R_i$ in terms of $\triangle \text{SNR}_i$ in the proper regime. Numerical examples for both equal weights and different weights are shown in Section V.

We notice that a fair or efficient allocation is Pareto optimal, i.e., no user's rate can be further increased without decreasing the rate of another user. However, an efficient or fair allocation need not fully utilize the resources at the relay, i.e., $\sum_{i \in \mathcal{I}} P_{r,d_i}$ can be less than P. This could happen, for example, when the relay is far away from all users so that allowing the relay to transmit half of the time will only decrease the total achievable rate. This is very different from most previous network resource allocation problems (e.g. [16]), in which the network performance is maximized only if the resource is fully utilized.

Since $\triangle R_i (P_{r,d_i})$ is non-smooth and non-concave, both Problems (13) and (14) are difficult to solve even in a centralized fashion. In the rest of the paper, we will propose two auction mechanisms, and quantify the technical conditions under which the auction mechanisms solve the above problems in a distributed fashion.

III. AUCTION MECHANISMS

An auction is a decentralized market mechanism for allocating resources. The essence of an auction is a game, where the players are the bidders, the strategies are the bids, and both allocations and payments are functions of the bids. One well known auction is the Vickrey-Clarke-Groves (VCG) auction [13], which requires gathering global information from the network and performing centralized computations. To overcome the limitation of the VCG auction, we propose

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[1]Most work on cooperative networks is based on information theoretic analysis, with the focus on calculating the achievable capacity (i.e., users' rate increases). Although we follow the same tradition in this paper, the proposed algorithms can be used to achieve other network objectives, such as α-fairness, that are more commonly used in the networking community.

two simple auctions, the *SNR auction* and the *power auction*, both of which have simple rules as described next.[2]

SNR Auction and Power Auction

- *Information*: Besides the public and local information (i.e., $W, P, \sigma^2, P_{s_i}, G_{s_i,d_i}$), each user i also knows the channel gains $G_{s_i,r}$ and G_{r,d_i}, through feedback from relay r. The relay announces a positive *reserve bid* $\beta > 0$ and a *price* $\pi > 0$ to all users before the auction starts.
- *Bids*: User i submits a scalar $b_i \geq 0$ to the relay.
- *Allocation*: The relay allocates transmit power according to

$$P_{r,d_i} = \frac{b_i}{\sum_{j \in \mathcal{I}} b_j + \beta} P. \qquad (16)$$

- *Payments*: In an SNR auction, source i pays the relay $C_i = \pi q_i \triangle \text{SNR}_i$, where q_i is a user-specific parameter. In a power auction, source i pays the relay $C_i = \pi P_{r,d_i}$.

A bidding profile is defined as the vector containing the users' bids, $\boldsymbol{b} = (b_1, ..., b_I)$. The bidding profile of user i's opponents is defined as $\boldsymbol{b}_{-i} = (b_1, ..., b_{i-1}, b_{i+1}, ..., b_I)$, so that $\boldsymbol{b} = (b_i; \boldsymbol{b}_{-i})$. User i chooses b_i to maximize its payoff

$$U_i (b_i; \boldsymbol{b}_{-i}, \pi) = \triangle R_i (P_{r,d_i} (b_i; \boldsymbol{b}_{-i})) - C_i (b_i; \boldsymbol{b}_{-i}, \pi). \qquad (17)$$

For notational simplicity, we omit the dependence on β and other system parameters.

If the reserve bid $\beta = 0$, then the resource allocation in (16) depends only on the ratio of the bids. A bidding profile $k\boldsymbol{b}$ (for any $k > 0$) leads to the same resource allocation as $\boldsymbol{b}$, which is not desirable in practice. That is why we need a positive reserve bid. However, the value of β is not important as long as it is positive. For example, if we increase β to $k'\beta$, then users can simply scale $\boldsymbol{b}$ to $k'\boldsymbol{b}$ (for any $k' > 0$), which leads to the same resource allocation. For simplicity, we choose $\beta = 1$ in all the simulations in Section V.

The desirable outcome of an auction is called a *Nash Equilibrium* (NE), which is a bidding profile $\boldsymbol{b}^*$ such that no user wants to deviate unilaterally, i.e.,

$$U_i \left(b_i^*; \boldsymbol{b}_{-i}^*, \pi\right) \geq U_i \left(b_i; \boldsymbol{b}_{-i}^*, \pi\right), \forall i \in \mathcal{I}, \forall b_i \geq 0. \qquad (18)$$

Define user i's *best response* (for fixed $\boldsymbol{b}_{-i}$ and price π) as

$$\mathcal{B}_i (\boldsymbol{b}_{-i}, \pi) = \left\{ b_i \left| b_i = \arg \max_{\tilde{b}_i \geq 0} U_i \left(\tilde{b}_i; \boldsymbol{b}_{-i}, \pi\right) \right. \right\}, \qquad (19)$$

which in general could be a set. An NE is also a fixed point solution of all users' best responses. We would like to answer the following four questions for both auctions: 1) When does an NE exist? 2) When is the NE unique? 3) What are the properties of the NE? 4) How can we reach the NE in a distributed fashion?

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[2]Both auctions are similar to the ones proposed in [16]. However, here we consider the problem of relay resource allocation in a multiple-hop wireless network, which is different from the sing-hop wireless network considered in [16]. For example, due to the unique characteristics of the relay network, especially the non-smooth and non-concave nature of the rate increase function (e.g., Fig. 2), calculating the best response functions here demands a completely new method and leads to a very different threshold policy. We also consider the multiple relay case in Section IV, whereas [16] considers the problem of allocating resource only at a single measurement point.
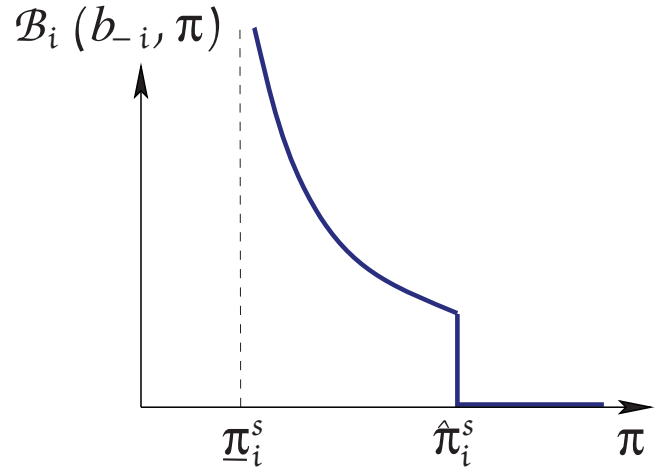

Fig. 3.   User i's best response in an SNR auction if $\underline{\pi}_i^s < \hat{\pi}_i^s$.

A. SNR Auction

Let us first study the users' best responses (e.g., (19)) in the SNR auction. For each user i, there exist two critical price values, $\underline{\pi}_i^s$ and $\hat{\pi}_i^s$, where

$$\underline{\pi}_i^s \triangleq \frac{W}{2 q_i \ln 2 \left(1 + \Gamma_{s_i,d_i} + \frac{P G_{r,d_i} P_{s_i} G_{s_i,r}}{\left(P_{s_i} G_{s_i,r} + P G_{r,d_i} + \sigma^2\right)\sigma^2} \right)}, \qquad (20)$$

and $\hat{\pi}_i^s$ is the smallest positive root of

$$g_i^s (\pi) \triangleq \pi q_i (1 + \Gamma_{s_i,d_i})$$
$$- \frac{W}{2} \left(\log_2 \left(\frac{2 \pi q_i \ln 2}{W} (1 + \Gamma_{s_i,d_i})^2 \right) + \frac{1}{\ln 2} \right). \qquad (21)$$

Function $g_i^s (\pi)$ represents the maximum payoff a user can achieve if it decides to use the relay. It is clear that when $g_i^s (\pi) < 0$, the user is better off by not using the relay. Both $\underline{\pi}_i^s$ and $\hat{\pi}_i^s$ can be calculated based on local information (i.e., no information exchange is needed among different users), and one could be larger than the other depending on the specific system parameters.

Theorem 1: In an SNR auction, user i's unique best response function is

$$\mathcal{B}_i (\boldsymbol{b}_{-i}, \pi) = f_i^s (\pi) \left(\sum_{j \neq i} b_j + \beta \right). \qquad (22)$$

If $\hat{\pi}_i^s > \underline{\pi}_i^s$, then (23). If $\hat{\pi}_i^s \leq \underline{\pi}_i^s$, then $f_i^s (\pi) = \infty$ for $\pi < \hat{\pi}_i^s$ and $f_i^s (\pi) = 0$ for $\pi \geq \hat{\pi}_i^s$.

Let us consider the intuition behind (22) and (23). First consider the case where $\hat{\pi}_i^s > \underline{\pi}_i^s$, in which case $\mathcal{B}_i (\boldsymbol{b}_{-i}, \pi)$ is illustrated in Fig. 3. The price $\hat{\pi}_i^s$ determines when it is beneficial for user i to use the relay. With a price larger than $\hat{\pi}_i^s$, user i cannot obtain a positive payoff from the auction no matter what bid it submits; thus it should simply use direct transmission and achieve a rate of R_{s_i,d_i}. As a result, $\mathcal{B}_i (\boldsymbol{b}_{-i}, \pi)$ is discontinuous at $\hat{\pi}_i^s$. When $\pi \in (\underline{\pi}_i^s, \hat{\pi}_i^s)$, user i wants to participate in the auction, and its best response depends how much other users bid ($\boldsymbol{b}_{-i}$). When the price is smaller than $\underline{\pi}_i^s$, user i becomes so aggressive that it demands

$$f_i^s(\pi) = \begin{cases} \infty, & \pi \le \underline{\pi}_i^s \\ \dfrac{\left(P_{s_i} G_{s_i,r} + \sigma^2\right)\sigma^2}{\dfrac{P G_{r,d_i} P_{s_i} G_{s_i,r}}{\frac{W}{2\pi q_i \ln 2} - 1 - \Gamma_{s_i,d_i}} - \left(P_{s_i} G_{s_i,r} + P G_{r,d_i} + \sigma^2\right)\sigma^2}, & \pi \in \left(\underline{\pi}_i^s, \hat{\pi}_i^s\right) \\ 0, & \pi \ge \hat{\pi}_i^s \end{cases} \quad . \quad (23)$$

a large SNR increase that cannot be achieved even if all the resource is allocated to it. This is reflected by an infinite bid in (23). Next we consider the second case where $\hat{\pi}_i^s \le \underline{\pi}_i^s$. In this case, user i either cannot obtain a positive payoff or cannot achieve the desired SNR increase, and thus the best response is either 0 or ∞. A detailed proof of Theorem 1 can be found in Appendix A.

Combining (16) and (22), we know that if an NE exist, the relay power allocated for user i is

$$P_{r,d_i}(\pi) = \frac{f_i^s(\pi)}{f_i^s(\pi) + 1} P, \qquad (24)$$

and $\sum_{i \in \mathcal{I}} \frac{f_i^s(\pi)}{f_i^s(\pi)+1} < 1$. The strict inequality is due to the positive reserve bid β, and later we will show that the resulting resource waste can be made arbitrarily small under appropriate conditions.

Next, we need to find the fixed point of all users' best responses, i.e., the NE. A trivial case would be $\hat{\pi}_i^s \le \underline{\pi}_i^s$ for all users i, in which case there exists a unique all-zero NE $\boldsymbol{b}^* = \boldsymbol{0}$ when $\pi \ge \max_i \hat{\pi}_i^s$. The more interesting case would be the following.

Definition 1: A network is *SNR-regular* if there exists at least one user i such that $\hat{\pi}_i^s > \underline{\pi}_i^s$.

In an SNR-regular network, at least one user is interested in using the relay. A randomly generated network with many users and a relay will be SNR-regular with high probability, unless all users have good direct gains or the relay is far away from the users.

Theorem 2: Consider an SNR auction in an SNR-regular network. There exists a threshold price π_{th}^s such that a unique NE exists if $\pi > \pi_{th}^s$; otherwise, no NE exists.

A proof of Theorem 2 is given in Appendix B. Unlike the results in [16], the unique NE in Theorem 2 might not be a continuous function of π, due to the discontinuity of the best response function as shown in Fig. 3. This has been observed in the simulation results described in Section V. In particular, the unique NE could be all zero for any price $\pi > \pi_{th}^s$, even if the network is SNR-regular.

It can be seen that the "weighted marginal utility equalization" property of a fair allocation (i.e., the constraint in Problem (14)) is satisfied at the NE of the SNR auction. However, there is always some "resource waste" due to the positive reserve bid β (i.e., some power will never be allocated to any user). Nevertheless, by choosing a price π larger than but very close to π_{th}^s, we could reduce the resource waste to a minimum and approximate the fair allocation. Formally, we define a reduced feasible set parameterized by δ as

$$\mathcal{P}_r^\delta \triangleq \left\{ \boldsymbol{P}_r \,\middle|\, \sum_{i \in \mathcal{I}} P_{r,d_i} \le P(1-\delta), P_{r,d_i} \ge 0, \forall i \in \mathcal{I} \right\}. \qquad (25)$$

Theorem 3: Consider an SNR auction in an SNR-regular network, where $f_i^s(\pi)$ is continuous at π_{th}^s for each user i, and positive and finite for at least one user. There exists a $\delta' > 0$ such that for any $0 < \delta < \delta'$, there exists a price $\pi^{s,\delta}$ under which the unique NE achieves the fair allocation $\boldsymbol{P}_r^{\text{fair}}$ with a reduced feasible set $\mathcal{P}_r^\delta$.

A proof is given in Appendix C. A sufficiently small δ makes sure that we deal with a regime in which $f_i^s(\pi)$ is continuous for all users. This is also desirable in practice since we want to minimize the amount of resources wasted.

Finally, we want to mention that the threshold price π_{th}^s is difficult to find analytically. This means that the price π needs to be adjusted through a "trial-and-error" process. In particular, the price needs to be increased when users' bids are too large and decreased when the resource waste due to β is too large. Since users' best responses in (23) are monotonically decreasing in price, a bi-section search of the price can lead to the desirable outcome.

B. Power Auction

For a power auction, we can also derive a closed-form solution for the best response functions, which are much more complicated compared with those for the SNR auction. To simplify the notations, we define $\tilde{P}_{s_i,r} \triangleq P_{s_i} G_{s_i,r}$. Furthermore, we define two functions ($z_i(\pi)$ and $g_i^p(\pi)$) and two critical prices ($\underline{\pi}_i^p$ and $\bar{\pi}_i^p$) in (26) to (29) (where (26) and (27) are located at the top of the following page):

$$\bar{\pi}_i^p \triangleq \frac{W P_{s_i} G_{r,d_i} G_{s_i,r}}{2\sigma^2 \ln 2 \left(\tilde{P}_{s_i,r} + \sigma^2\right)\left(1 + \Gamma_{s_i,d_i}\right)}, \qquad (28)$$

and

$$g_i^p(\pi) = \frac{W}{2} \log_2\left(1 + \Gamma_{s_i,d_i} + \frac{z_i(\pi)\tilde{P}_{s_i,r}}{\sigma^2\left(z_i(\pi) + \tilde{P}_{s_i,r} + \sigma^2\right)}\right) - R_{s_i,d_i} - \pi \frac{z_i(\pi)}{G_{r,d_i}}. \qquad (29)$$

Proposition 1: $g_i^p(\pi)$ is strictly decreasing in $\pi \in [\underline{\pi}_i^p, \bar{\pi}_i^p]$.

A proof is given in Appendix D. It can also be shown that $g_i^p(0) = \frac{W}{2} \log_2\left(1 + \Gamma_{s_i,d_i} + \frac{P_{s_i} G_{s_i,r}}{\sigma^2}\right) - R_{s_i,d_i}$ and $g_i^p(\bar{\pi}_i^p) < 0$. If $g_i^p(0) > 0$, then there exists a unique root for $g_i^p(\pi)$ within $(0, \bar{\pi}_i^p]$, and we denote it as $\hat{\pi}_i^p$. If $g_i^p(0) \le 0$, we can simply let $\hat{\pi}_i^p = 0$. Then we have the following result:

Theorem 4: In a power auction, user i's unique best response function is

$$\mathcal{B}_i(\boldsymbol{b}_{-i}, \pi) = f_i^p(\pi)\left(\sum_{j \ne i} b_j + \beta\right). \qquad (30)$$

$$z_{1,i} = (\tilde{P}_{s_i,r} + \sigma^2)\left(\tilde{P}_{s_i,r} + 2\sigma^2\left(1 + \Gamma_{s_i,d_i}\right)\right),$$

$$z_{2,i}(\pi) = 4\left(\tilde{P}_{s_i,r} + \sigma^2\left(1 + \Gamma_{s_i,d_i}\right)\right)\left(\tilde{P}_{s_i,r} + \sigma^2\right)\left(\frac{W}{2\pi \ln 2}\tilde{P}_{s_i,r}G_{r,d_i} - \sigma^2\left(\tilde{P}_{s_i,r} + \sigma^2\right)\left(1 + \Gamma_{s_i,d_i}\right)\right),$$

$$z_i(\pi) = \frac{-z_{1,i} + \sqrt{z_{1,i}^2 + z_{2,i}(\pi)}}{2\left(\tilde{P}_{s_i,r} + \sigma^2\left(1 + \Gamma_{s_i,d_i}\right)\right)}. \tag{26}$$

$$\underline{\pi}_i^p \triangleq \frac{W\tilde{P}_{s_i,r}G_{r,d_i}\left(P_{s_i}G_{s_i,r} + \sigma^2\right)}{2\ln 2\left[\sigma^2\left(1 + \Gamma_{s_i,d_i}\right)\left(PG_{r,d_i} + \tilde{P}_{s_i,r} + \sigma^2\right) + \tilde{P}_{s_i,r}PG_{r,d_i}\right]\left(PG_{r,d_i} + \tilde{P}_{s_i,r} + \sigma^2\right)}, \tag{27}$$

If $\hat{\pi}_i^p > \underline{\pi}_i^p$, then

$$f_i^p(\pi) = \begin{cases} \infty, & \pi < \underline{\pi}_i^p \\ \frac{z_i(\pi)}{PG_{r,d_i} - z_i(\pi)}, & \pi \in [\underline{\pi}_i^p, \hat{\pi}_i^p] \\ 0, & \pi > \hat{\pi}_i^p \end{cases} . \tag{31}$$

If $\hat{\pi}_i^p \leq \underline{\pi}_i^p$, then $f_i^p(\pi) = \infty$ for $\pi < \hat{\pi}_i^p$ and $f_i^p(\pi) = 0$ for $\pi \geq \hat{\pi}_i^p$.

The proof of Theorem 4 is similar to that of Theorem 1 and thus is omitted due to space limitations. Theorem 4 is applicable for the general SNR regime. In the case of low SNR, i.e., when Γ_{s_i,d_i} and $\triangle\text{SNR}_i(b_i, \boldsymbol{b}_{-i})$ are small for all i, we have

$$W\log_2\left(1 + \Gamma_{s_i,d_i} + \triangle\text{SNR}_i(b_i, \boldsymbol{b}_{-i})\right)$$
$$\approx \frac{W}{\ln 2}\left(\Gamma_{s_i,d_i} + \triangle\text{SNR}_i(b_i, \boldsymbol{b}_{-i})\right). \tag{32}$$

In this case, we can show that $\hat{\pi}_i^p = 0$ when $G_{s_i,d_i} > G_{s_i,r}$. In other words, when the direct channel gain between source and destination is larger than the channel gain between source and relay, a user will never use the relay in the low SNR regime.

In terms of the existence, uniqueness and properties of the NE, we have the following results, which are similar in spirit to Theorems 2 and 3 for the SNR auction. The detailed proofs are omitted.

Definition 2: A network is *power-regular* if $\hat{\pi}_i^p > \underline{\pi}_i^p$ for at least one user i.

Theorem 5: Consider a power auction in a power-regular network. There exists a threshold price $\pi_{th}^p > 0$ such that a unique NE exists if $\pi > \pi_{th}^p$; otherwise, no NE exists.

Theorem 6: Consider a power auction in a power-regular network, where $f_i^p(\pi)$ is continuous at π_{th}^p for each user i, and positive and finite for at least one user. There exists a $\delta' > 0$ such that for any $0 < \delta < \delta'$, there exists a price $\pi^{p,\delta}$ under which the unique NE achieves the efficient allocation $\boldsymbol{P}_r^{\text{efficient}}$ with a reduced feasible set $\mathcal{P}_r^\delta$.

We can see that for both auctions, the conditions for the existence, uniqueness, and effectiveness (fairness for SNR auction and efficiency for power auction) are quite similar. This is due to the linearity of the best response functions in both cases, which again is due to the weighted proportional allocation rule of the relay power in (16).

C. Distributed Iterative Best Response Updates

The last issue we want to address is how the NE can be reached in a distributed fashion. It is clear that the best response function in (23) can be calculated in a distributed fashion with limited information feedback from the relay. However, each user does not have enough information to calculate the best responses of other users, which prevents it from directly calculating the NE. Nevertheless, the NE can be achieved in a distributed fashion if we allow the users to *iteratively* update their bids based on best response functions in an asynchronous fashion.

Let us first consider the synchronous updates, where users update their bids $\boldsymbol{b}(t)$ at time t according to the best response functions $b_i(t) = \mathcal{B}_i(\boldsymbol{b}_{-i}(t-1), \pi)$, based on other users' bids $\boldsymbol{b}_{-i}(t-1)$ at time $t-1$. Taking the SNR auction as an example, each user updates according to

$$b_i(t) = f_i^s(\pi)\left(\sum_{j \neq i} b_j(t-1) + \beta\right). \tag{33}$$

To implement (33) directly, each user i needs to know the sum of all other users' bids, which may not be possible in practice. However, we can show that (33) can be written in an equivalent form as

$$b_i(t) = f_i^{s,d}(\pi, t-1)b_i(t-1), \tag{34}$$

where (35) applies. It can be seen that $f_i^{s,d}(\pi)$ can be calculated locally by user i. When $\pi \in [\underline{\pi}_i^s, \hat{\pi}_i^s]$, $f_i^{s,d}(\pi)$ depends on the difference between $\triangle\text{SNR}_i(b_i(t-1), \boldsymbol{b}_{-i}(t-1))$ and $\left(\frac{W}{2\pi \ln 2} - 1 - \Gamma_{s_i,d_i}\right)$, and $f_i^{s,d}(\pi)$ does not equal 1 unless the two terms are the same. For the power auction, we can similarly show that the best response update can be written as

$$b_i(t) = f_i^{p,d}(\pi, t-1)b_i(t-1), \tag{36}$$

where $f_i^{p,d}(\pi, t-1)$ is similar to (35) (with more complicated notation), and is also locally computable.

In practice, users may not update their bids in each time slot. Therefore, we need to prove the convergence of the algorithm under *asynchronous* updates. The complete asynchronous best response update algorithm is given in Algorithm 1[3], where

---

[3] $[x]_a^b = \max\{\min\{x, b\}, a\}$. $f_i^d = f_i^{s,d}$ or $f_i^{p,d}$ depending on which auction the users participate in.

$$f_i^{s,d}(\pi, t-1) = \begin{cases} \infty, & \pi < \underline{\pi}_i^s \\ \frac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\triangle \text{SNR}_i(b_i(t-1), \boldsymbol{b}_{-i}(t-1))} - \sigma^2(PG_{r,d_i} + P_{s_i}G_{s_i,r} + \sigma^2), & \pi \in [\underline{\pi}_i^s, \hat{\pi}_i^s] \\ \frac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\frac{W}{2\pi q_i \ln 2} - 1 - \Gamma_{s_i,d_i}} - \sigma^2(PG_{r,d_i} + P_{s_i}G_{s_i,r} + \sigma^2), & \\ 0, & \pi > \hat{\pi}_i^s \end{cases} \quad (35)$$

each user i updates its bid only if the current time slot belongs to a set $\mathcal{T}_i$, which is an unbounded set of time slots. For example, $\mathcal{T}_i = \{1, 3, 7, ...\}$ means that user i updates b_i at time slot 1, 3, 7, etc. We make a very mild assumption that the asynchronism of the updates is bounded, i.e., there exists a finite but sufficiently large positive constant B (called the *asynchronism measure* [17, pp. 481]) such that

$$\forall t_1 \in \mathcal{T}_i, \ \exists t_2 \in \mathcal{T}_i, \ t_2 - t_1 \le B, \forall i \in \mathcal{I}. \quad (37)$$

From (37), each user updates its bid at least once during any time interval of length B slots. The exact value of B is not important and need not be known by the users.

Algorithm 1 Asynchronous Best Response Updates of Bids

1: $t = 0$.
2: Each user i randomly chooses a feasible $b_i(0) \in [\underline{b}_i, \bar{b}_i]$.
3: $t = t + 1$.
4: **for** each user $i \in \mathcal{I}$
5:   **if** $t \in \mathcal{T}_i$ **then**
6:    $b_i(t) = \left[f_i^d(\pi, t-1) b_i(t-1)\right]_{\underline{b}_i}^{\bar{b}_i}$.
7:   **end if**
8: **end for**
9: Go to Line 3.

Theorem 7: If there exists a unique nonzero NE in the SNR or power auction, there always exists a lowerbound bid vector $\underline{\boldsymbol{b}} = \{\underline{b}_i\}_{i \in \mathcal{I}}$ and an upperbound bid vector $\bar{\boldsymbol{b}} = \{\bar{b}_i\}_{i \in \mathcal{I}}$, under which Algorithm 1 globally converges to the unique NE.

In practice, it is sufficient to choose $\underline{\boldsymbol{b}}$ to be a sufficiently small and positive vector and $\bar{\boldsymbol{b}}$ to be a sufficiently large and finite vector. To prove Theorem 7, we can first verify that the best response update in (33) is in fact a *standard interference function* [19], and the rest of the proof is similar to that of Theorem 4 in [19].

IV. EXTENSIONS TO MULTIPLE RELAY NETWORKS

The proposed auction-based resource allocation algorithms can be generalized to networks with multiple relays. Let us define the set of relays as $\mathcal{K} = \{1, ..., K\}$. Each relay $k \in \mathcal{K}$ announces a price π_k and a reserve bid β_k, without knowing the prices and reserve bids of other relays. Each user $i \in \mathcal{I}$ submits a nonnegative bid vector $\boldsymbol{b}_i = \{b_{ik}\}_{k \in \mathcal{K}}$, one component for each relay. Based on the bids, relay k allocates user i with transmission power

$$P_{r_k, d_i} = \frac{b_{ik}}{\sum_{j \in \mathcal{I}} b_{jk} + \beta_k} P_k, \quad (38)$$

where P_k is the fixed total transmission power of relay k. This leads to an SNR increase of user i of

$$\triangle \text{SNR}_{ik} = \Gamma_{s_i, r_k, d_i} = \frac{P_{r_k, d_i} P_{s_i} G_{r_k, d_i} G_{s_i, r_k}}{\sigma^2(P_{r_k, d_i} G_{r_k, d_i} + P_{s_i} G_{s_i, r_k} + \sigma^2)}. \quad (39)$$

The total information rate that user i achieves is

$$R_{s_i, \boldsymbol{r}, d_i} = \frac{W \log_2(1 + \Gamma_{s_i, d_i} + \sum_k \triangle \text{SNR}_{ik})}{\sum_k \mathbf{1}_{\{\triangle \text{SNR}_{ik} > 0\}} + 1}. \quad (40)$$

This includes a special case in which user i does not use any relay (i.e., $\triangle \text{SNR}_{ik} = 0$ for all $k \in \mathcal{K}$). It is clear that $\triangle \text{SNR}_{ik} > 0$ if and only if $b_{ik} > 0$. Notice that using more relays leads to higher total SNR increase $\sum_k \triangle \text{SNR}_{ik}$, but also leads to a smaller coefficient $1/(\sum_k \mathbf{1}_{\{\triangle \text{SNR}_{ik} > 0\}} + 1)$. User i needs to compare the total achievable rate under difference choices of relays, and choose one that yields the highest rate increase.

We first consider the SNR auction where user i's payment is $C_i = \sum_k \pi_k q_{ik} \triangle \text{SNR}_{ik}$.

Theorem 8: In an SNR auction with multiple relays, a user i either does not use any relay, or only uses one relay $r_{k(i)}$ with the smallest weighted price, i.e., $k(i) = \arg\min_{k \in \mathcal{K}} \pi_k q_{ik}$.

Theorem 8 can be proved by examining the first order conditions that need to be satisfied at the NE. In case there are multiple relays that announce the same smallest weighted price, user i can randomly choose one of them. The choice of relay can be made by a user before knowing other users' bids. This implies that we can divide a multiple-relay network into $K + 1$ clusters of nodes: each of the first K clusters contains one relay node and the users who use this relay, and the last cluster contains users that do not use any relay. Then we can analyze each cluster independently as a single-relay network as in Section III-A.

Now consider the power auction where user i's payment is $C_i = \sum_k \pi_k P_{r_k, d_i}$. There are several key differences here compared with the SNR auction. First, a user may choose to use multiple relays simultaneously here. User i's best response can be written in the following linear form:

$$\mathcal{B}_{i,k}(\boldsymbol{b}_{-i,k}, \boldsymbol{\pi}) = f_{i,k}^p(\boldsymbol{\pi}) \left(\sum_{j \ne i} b_{j,k} + \beta_k\right), \forall k \in \mathcal{K}. \quad (41)$$

There are two key differences here compared with the single-relay power auction. First, to calculate $f_{i,k}^p(\boldsymbol{\pi})$, user i needs to consider a total of $\sum_{l=0}^K \binom{K}{l}$ cases of choosing relays. For example, when there are two relays in the network, a user needs to consider four cases: not using any relay, using relay 1 only, using relay 2 only, and using both relays. For the given relay choice in case n, it calculates the linear coefficients $f_{i,k}^{p,n}(\boldsymbol{\pi})$ for all k in closed-form (also involves threshold policy similar as in the SNR auction) and the
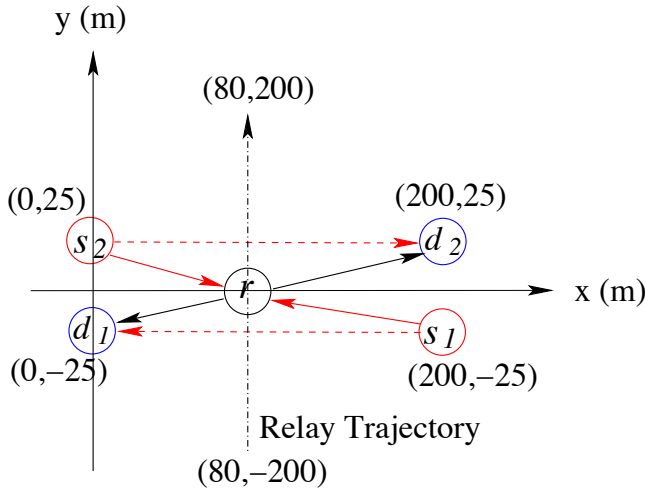
Fig. 4.   A two-user cooperative network

Fig. 5.   Individual rate increase vs. relay location (y-axis) for an SNR auction and a power auction (users have equal weights in the SNR auction).

corresponding rate increase $\triangle R_i^n$. Then it finds the case that yields the largest payoff, $n^* = \arg\max_n \triangle R_i^n$, and sets $f_{i,k}^p(\boldsymbol{\pi}) = f_{i,k}^{p,n^*}(\boldsymbol{\pi}) \; \forall k$. Second, the linear coefficient $f_{i,k}^p(\boldsymbol{\pi})$ is dependent on the prices announced by all relays. For example, a large π_k or a small $\pi_{k'}$ ($k' \neq k$) may lead to $f_{i,k}^p(\boldsymbol{\pi}) = 0$, i.e., user i chooses not to use relay r_k.

Similar to the single relay power auction, we can also calculate a threshold price $\pi_{k,th}^p$ for relay r_k. In this case, we assume that all relays except r_k announce infinitely high prices, and then calculate $\pi_{k,th}^p$ such that $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^p(\pi_k)}{f_{i,k}^p(\pi_k)+1} < 1$ when $\pi_k > \pi_{k,th}^p$, and $\sum_{i \in \mathcal{I}} \frac{f_{i,k}^p(\pi_k)}{f_{i,k}^p(\pi_k)+1} \geq 1$ when $\pi_k \leq \pi_{k,th}^p$.

Corollary 1: In a power auction with multiple relays, there exists an NE if $\pi_k > \pi_{k,th}^p$ for each k.

On the other hand, necessary condition for the existence of an NE as well as conditions for uniqueness are not straightforward to specify, and are left for future research.

Finally, note that if there exists a unique NE in the SNR auction or power auction with multiple relays, the asynchronous best response updates of the users will globally converge to that NE.

V. SIMULATION RESULTS

For illustration purposes, here we provide simulation results for a single relay network. We first simulate two auction mechanisms for a two-user network. As shown in Fig. 4, the locations of the two sources (s_1 and s_2) and two destinations (d_1 and d_2) are fixed at (200m,-25m), (0m,25m), (0m,-25m), (200m,25m). The x coordinate of the relay node r at 80m, and its y coordinate varies within the range [-200m,200m]. In the simulation, the relay moves along the dotted line. The propagation loss factor is set to 4, and the channel gains are distance based (i.e., time-varying fading is not considered here). The transmit power between a source and its destination is $P_i = 0.01$W, the noise level is $\sigma^2 = 10^{-11}$W, and the total power of the relay node is $P = 0.1$W.

In Fig. 5, we show the individual rate increases of both users in both auctions. The weights in the SNR auction are chosen as $q_i = 1$ for each user i. We first consider the power auction. The individual rate increases under the power auction are similar as
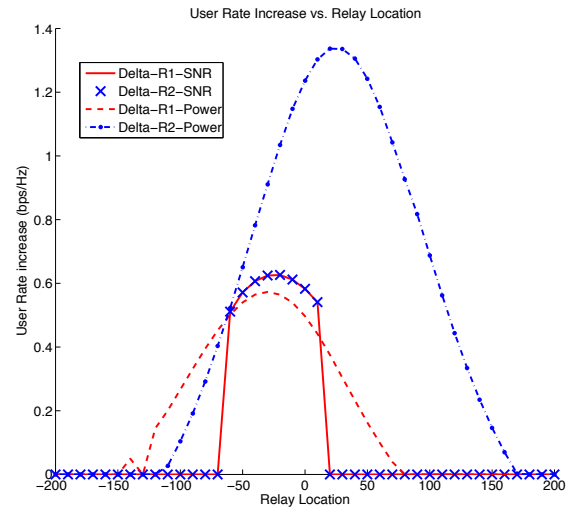
that achieved under a VCG auction (not shown here) and thus are efficient. Since the relay movement trajectory is relatively closer to source s_2 than to source s_1, user 2 achieves an overall better performance compared with user 1. In particular, user 2 achieves a peak rate increase of 1.35 bps/Hz when the relay is at location 25m (y-axis), compared with the peak rate increase of 0.56 bps/Hz achieved by user 1 when the relay is at location -25m. Things are very different in the SNR auction, where the resource allocation is fair. In particular, since the distance between a source and its destination is the same for each of the users, both users achieve the same positive rate increases when they both use the relay. This is the case when the relay is between locations -60m and 10m. At other locations, users simply choose not to use the relay since they cannot both get equal rate increase while obtaining a positive payoff. This shows the tradeoff between efficiency and fairness.

In Fig. 6, we show the individual rate increases for two auctions, with the weights in the SNR auction set to $q_1 = 0.5$ and $q_2 = 1$. Here user 2 achieves a higher rate increase than user 1 in the SNR auction, due to its higher weight. By adjusting the weights of different users, the SNR auction can achieve different tradeoffs between fairness and efficiency.

Next, we consider the case where there are 20 users in the network, with their source nodes and destination nodes randomly and uniformly located within the square field that has the same range of [-150m,150m] on both the x-axis and the y-axis. A single relay is fixed at the location (0m,0m). We change the total transmission power P of the relay from 0.04 W to 1 W. Figs. 7 and 8 show the corresponding simulation results. Each point in the figures represents the averaged result over 100 randomly generated network topologies. All users have the same weights in SNR auction. With an increasing amount of resource at the relay node, the total network rate increase improves in both auctions (Fig. 7), and the power auction achieves higher rate increase than the SNR auction. Fig. 8 shows the variance of the rate increase (among the users with positive rate increase), and it is clear that the SNR
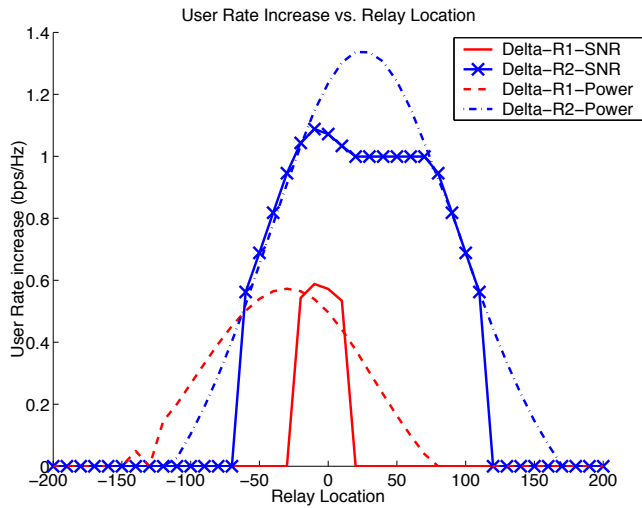
Fig. 6.   Individual rate increase vs. relay location (y-axis) for an SNR auction and a power auction (users have different weights in the SNR auction).
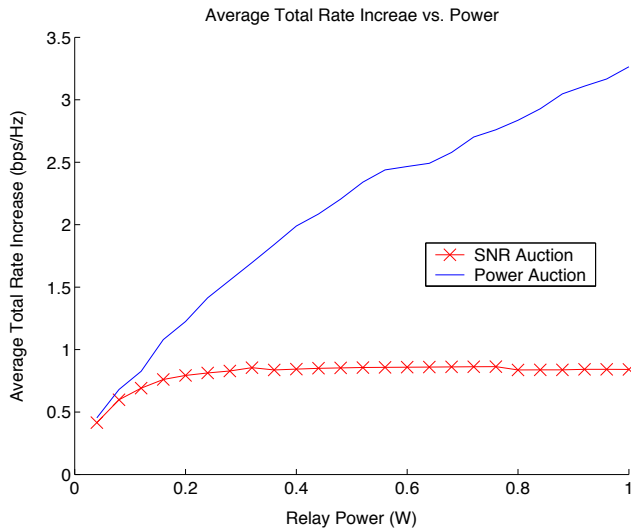

Fig. 8.   Total positive rate increase variance vs. relay power.

Fig. 7.   Total network rate increase vs. relay power.
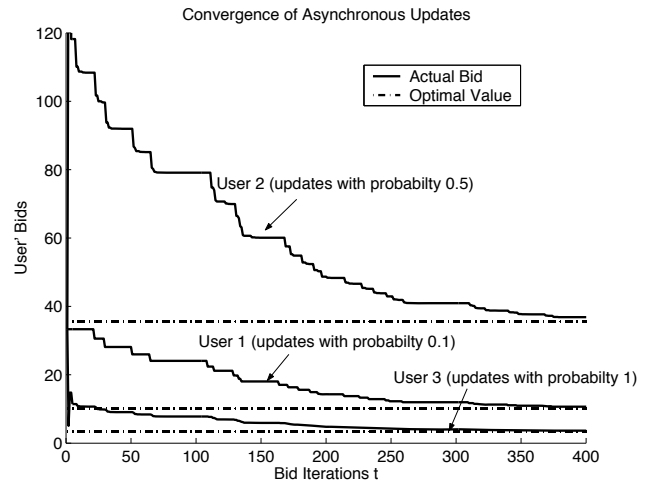
Fig. 9.   Convergence of bids under asynchronous updates.

auction achieves a fairer resource allocation as indicated by the almost zero variance in all cases.

Finally, we show the convergence of Algorithm 1. We consider a three user network, where the three transmitters are located at (100m,-25m), (-100m,25m) and (100m,5m), and the three receivers are located at (-100m,25m), (100m,25m) and (-100m,5m). The relay is located at (0m,-2m). We simulate the SNR auction in this case. Three users randomly and independently choose to update their own bids in each time slot with probability 0.1, 0.5 and 1, respectively. Convergence is clearly observed.

VI. CONCLUSIONS

Cooperative transmission can greatly improve communication system performance by taking advantage of the broadcast nature of wireless channels and cooperation among users. In this paper, we have proposed two auction mechanisms, the SNR auction and the power auction, to distributively coordinate the relay power allocation among users. Under
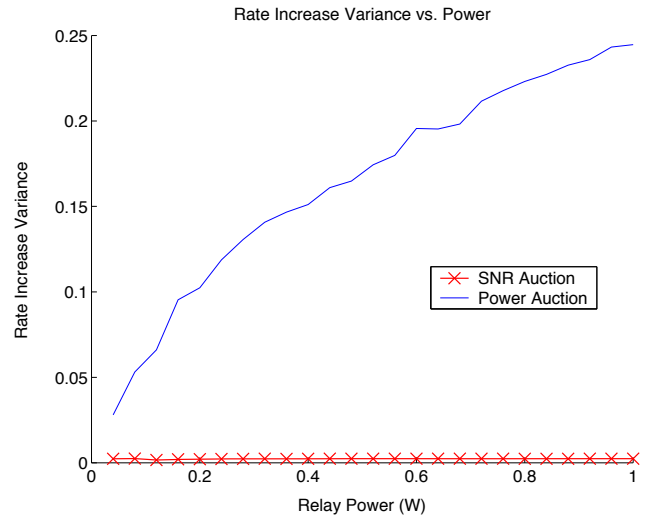
a fixed price announced by a single relay, we have shown that although each user has a non-smooth, non-concave utility function, its best response function can nevertheless explicitly be calculated locally based on a simple threshold policy. The existence and uniqueness of the Nash equilibrium in both auctions has been proved using nonnegative matrix theory. Furthermore, under properly chosen prices, the power auction has been shown to achieve the efficient allocation, and the SNR auction has been seen to be flexible in achieving various tradeoffs between fairness and efficiency depending on the priority weights. Furthermore, we have shown that users can achieve the unique NE in a completely distributed and asynchronous fashion. We have also shown that the main properties of the two auctions are applicable for networks with multiple relays at different locations.

APPENDIX

A. Proof of Theorem 1

In an SNR auction, user i's payoff function is

$$U_i\left(b_i, \boldsymbol{b}_{-i}, \pi\right) = \triangle R_i\left(P_{r,d_i}\left(b_i; \boldsymbol{b}_{-i}\right)\right) - \pi q_i \triangle \mathrm{SNR}_i\left(b_i, \boldsymbol{b}_{-i}\right), \tag{42}$$

$$\triangle \quad R_i\left(P_{r,d_i}\left(b_i;\boldsymbol{b}_{-i}\right)\right) \quad = \quad \max\left\{\frac{W}{2}\log_2\left(1+\Gamma_{s_i,d_i}+\triangle\,\mathrm{SNR}_i\left(b_i,\boldsymbol{b}_{-i}\right)\right)-R_{s_i,d_i},0\right\} \quad (43)$$

where (43) is true. The best response function is

$$\mathcal{B}_i\left(\boldsymbol{b}_{-i},\pi\right)=\arg\max_{b_i\geq 0}U_i\left(b_i,\boldsymbol{b}_{-i},\pi\right). \quad (44)$$

The difficulty of finding $\mathcal{B}_i\left(\boldsymbol{b}_{-i},\pi\right)$ is due mainly to the $\max$ operation in (43), which leads to non-smoothness and non-concavity of the payoff function. Next, we solve for $\mathcal{B}_i\left(\boldsymbol{b}_{-i},\pi\right)$ in two steps.

First, by temporarily ignoring the $\max$ operation, we solve for the following *modified best response function*,

$$\tilde{\mathcal{B}}_i\left(\boldsymbol{b}_{-i},\pi\right)=\arg\max_{b_i\geq 0}\tilde{U}_i\left(b_i,\boldsymbol{b}_{-i},\pi\right), \quad (45)$$

where the *modified payoff function* is

$$\begin{aligned}\tilde{U}_i\left(b_i,\boldsymbol{b}_{-i},\pi\right)=&\frac{W}{2}\log_2\left(1+\Gamma_{s_i,d_i}+\triangle\,\mathrm{SNR}_i\left(b_i,\boldsymbol{b}_{-i}\right)\right)\\&-R_{s_i,d_i}-\pi q_i\triangle\,\mathrm{SNR}_i\left(b_i,\boldsymbol{b}_{-i}\right).\end{aligned} \quad (46)$$

Differentiating $\tilde{U}_i\left(b_i,\boldsymbol{b}_{-i},\pi\right)$ with respect to b_i, we have

$$\begin{aligned}\frac{\partial \tilde{U}_i\left(b_i,\boldsymbol{b}_{-i},\pi\right)}{\partial b_i}=&\frac{\partial\triangle\,\mathrm{SNR}_i\left(b_i,\boldsymbol{b}_{-i}\right)}{\partial b_i}\\&\times\left(\frac{W}{2\ln 2\left(1+\Gamma_{s_i,d_i}+\triangle\,\mathrm{SNR}_i\left(b_i,\boldsymbol{b}_{-i}\right)\right)}-\pi q_i\right).\end{aligned} \quad (47)$$

It is clear that $\partial\triangle\,\mathrm{SNR}_i\left(b_i,\boldsymbol{b}_{-i}\right)/\partial b_i$ is strictly positive. Furthermore, the term in the brackets on the right hand side of (47) is strictly decreasing in b_i. If the price is appropriate such that there exists a b_i so that the term in the brackets equals zero, we have found the best response. If the price is too high (or too low), the best response is 0 (or ∞). In summary, we can show that the unique best response $\tilde{\mathcal{B}}_i\left(\boldsymbol{b}_{-i},\pi\right)$ in this case can be written as

$$\tilde{\mathcal{B}}_i\left(\boldsymbol{b}_{-i},\pi\right)=\tilde{f}_i^s\left(\pi\right)\left(\sum_{j\neq i}b_j+\beta\right), \quad (48)$$

where the equation on the top of the following page is true. $\underline{\pi}_i^s$ is defined in (20) and $\bar{\pi}_i^s=1/(2q_i\ln 2\left(1+\Gamma_{s_i}^{DT}\right))>\underline{\pi}_i^s$. As a result, we can calculate the maximum value of the modified payoff function, $\tilde{U}_i\left(\tilde{\mathcal{B}}_i\left(\boldsymbol{b}_{-i},\pi\right),\boldsymbol{b}_{-i},\pi\right)$, which turns out to depend only on the price π,

$$\tilde{U}_i^{\max}\left(\pi\right)=\begin{cases}g_i^s\left(\underline{\pi}_i^s\right), & \pi<\underline{\pi}_i^s\\g_i^s\left(\pi\right), & \pi\in[\underline{\pi}_i^s,\bar{\pi}_i^s]\\g_i^s\left(\bar{\pi}_i^s\right), & \pi>\bar{\pi}_i^s\end{cases} \quad (49)$$

where the function $g_i^s\left(\cdot\right)$ is defined in (21).

Next, we consider the $\max$ operation in (43) and consider the original payoff function $U_i\left(b_i;\boldsymbol{b}_{-i},\pi\right)$ as in (42). We notice that user i can always submit a zero bid and achieve a zero payoff. This implies that the difference between the best response function $\mathcal{B}_i\left(\boldsymbol{b}_{-i},\pi\right)$ and the modified one $\tilde{\mathcal{B}}_i\left(\boldsymbol{b}_{-i},\pi\right)$ occurs in the prices range where $\tilde{U}_i^{\max}\left(\pi\right)<0$. It can be verified that $g_i^s\left(\pi\right)$ is strictly decreasing over $(0,\bar{\pi}_i^s]$,

with $g_i^s\left(\bar{\pi}_i^s\right)<0$ and $\lim_{\pi\to 0}g_i^s\left(\pi\right)>0$. Thus using bisection search, we can find a unique price $\hat{\pi}_i\in(0,\bar{\pi}_i^s)$ such that $g_i^s\left(\hat{\pi}_i^s\right)=0$. Then for any price $\pi>\hat{\pi}_i^s$, we have $\tilde{U}_i^{\max}\left(\pi\right)<0$, which implies that $\mathcal{B}_i\left(\boldsymbol{b}_{-i},\pi\right)=0$. Together with the result in (49), we have proved the theorem. ∎

B. Proof of Theorem 2

Based on (22), we know that an NE $\boldsymbol{b}^*$ is a nonnegative solution of the following set of linear equations:

$$\left(\boldsymbol{I}-\boldsymbol{F}^s\left(\pi\right)\right)\boldsymbol{b}=\boldsymbol{f}^s\left(\pi\right)\beta, \quad (50)$$

where $\boldsymbol{I}$ is an $I\times I$ identity matrix, $\boldsymbol{F}^s\left(\pi\right)$ is an $I\times I$ matrix whose (i,j)th component equals to $f_i^s\left(\pi\right)$ for $i\neq j$, and diagonal terms equal to zero, and $\boldsymbol{f}^s\left(\pi\right)$ is an $I\times 1$ vector whose ith component equals $f_i^s\left(\pi\right)$. It is clear that there exists a unique NE if and only if the spectral radius of the nonnegative matrix $\boldsymbol{F}^s\left(\pi\right)$ (defined as $\rho\left(\boldsymbol{F}^s\left(\pi\right)\right)$) is less than 1, in which case the unique NE is $\boldsymbol{b}^*=\sum_{n=0}^{\infty}\left(\boldsymbol{F}^s\left(\pi\right)\right)^n\boldsymbol{f}^s\left(\pi\right)\beta$. By using nonnegative matrix theory [18], we can show that $\rho\left(\boldsymbol{F}^s\left(\pi\right)\right)$ is a decreasing function of π, and we can find a threshold π_{th}^s such that $\rho\left(\boldsymbol{F}^s\left(\pi\right)\right)<1$ when $\pi>\pi_{th}^s$, and $\rho\left(\boldsymbol{F}^s\left(\pi\right)\right)\geq 1$ otherwise. Details are omitted due to space limitations. ∎

C. Proof of Theorem 3

Here we follow the notations used in the proof of Theorem 2. Since $f_i^s\left(\pi\right)$ is continuous at π_{th}^s for all users, we can show that $\rho\left(\boldsymbol{F}^s\left(\pi\right)\right)$ is also continuous at π_{th}^s. Moreover, since $f_i^s\left(\pi\right)$ is positive and finite for at least one user, we can show that $\rho\left(\boldsymbol{F}^s\left(\pi\right)\right)$ can be made arbitrarily close but smaller than 1 with properly chosen price. This means that the summation $\sum_i b_i^*$ (where $\boldsymbol{b}^*=\sum_{n=0}^{\infty}\left(\boldsymbol{F}^s\left(\pi\right)\right)^n\boldsymbol{f}^s\left(\pi\right)\beta$ is the unique NE) can be made arbitrarily large, and the resource waste factor $\beta P/\left(\sum_i b_i^*+\beta\right)$ can be made arbitrarily small. ∎

D. Proof of Proposition 1

Let us define the modified payoff function as

$$\begin{aligned}\tilde{U}_i\left(P_{r,d_i},\pi\right)=&\frac{W}{2}\log_2\left(1+\Gamma_{s_i,d_i}+\triangle\,\mathrm{SNR}_i\left(P_{r,d_i}\right)\right)\\&-R_{s_i,d_i}-\pi P_{r,d_i}.\end{aligned} \quad (51)$$

We can show that the unique value of P_{r,d_i} that maximizes $\tilde{U}_i\left(P_{r,d_i}\right)$ is

$$P_{r,d_i}^*\left(\pi\right)=\arg\max_{P_{r,d_i}\geq 0}\tilde{U}_i\left(P_{r,d_i},\pi\right)=\frac{z_i\left(\pi\right)}{G_{r,d_i}},\forall\pi\in[\underline{\pi}_i^p,\bar{\pi}_i^p], \quad (52)$$

with $P_{r,d_i}^*\left(\underline{\pi}_i^p\right)=P$ and $P_{r,d_i}^*\left(\bar{\pi}_i^p\right)=0$. Furthermore, $g_i^p\left(\pi\right)=\tilde{U}_i\left(P_{r,d_i}^*\left(\pi\right),\pi\right)$.
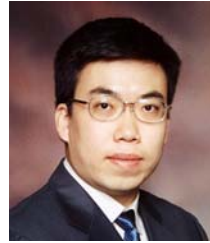
Assume that $g_i^p\left(\pi\right)$ is not strictly decreasing with $\pi\in[\underline{\pi}_i^p,\bar{\pi}_i^p]$. Then there exists a $\pi'<\pi''$, such that $g_i^p\left(\pi'\right)\leq$

$$\tilde{f}_i^s(\pi) = \begin{cases} \infty, & \pi < \underline{\pi}_i^s \\ \dfrac{\left(P_{s_i}G_{s_i,r}+\sigma^2\right)\sigma^2}{\dfrac{PG_{r,d_i}P_{s_i}G_{s_i,r}}{\frac{W}{2\pi q_i \ln 2}-1-\Gamma_{s_i,d_i}}-\left(P_{s_i}G_{s_i,r}+PG_{r,d_i}+\sigma^2\right)\sigma^2}, & \pi \in [\underline{\pi}_i^s, \bar{\pi}_i^s] \\ 0, & \pi > \bar{\pi}_i^s \end{cases}$$

$g_i^p(\pi'')$, i.e, $\tilde{U}_i\left(P_{r,d_i}^*(\pi'),\pi'\right) \leq \tilde{U}_i\left(P_{r,d_i}^*(\pi''),\pi''\right)$. However, from (51) and the optimality of $P_{r,d_i}^*(\pi)$ we know that $\tilde{U}_i\left(P_{r,d_i}^*(\pi'),\pi'\right) \geq \tilde{U}_i\left(P_{r,d_i}^*(\pi''),\pi'\right) > \tilde{U}_i\left(P_{r,d_i}^*(\pi''),\pi''\right)$, which leads to a contradiction. This completes the proof. ∎

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