An Iterative Interference Alignment Algorithm for the General MIMO X Channel

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***Abstract*— Interference alignment (IA) has been shown as an important technique to achieve a linear capacity scaling in wireless communications. However, the IA scheme over finite signaling dimensions for a general multiple-input-multiple-output (MIMO) X channel is still rarely studied. The main challenge of MIMO X channels is that the two sets of conditions for IA, namely the interference nulling conditions and the rank preservation conditions, get coupled. The usual IA methods for the interference channel and the broadcasting channel cannot be applied anymore. In this paper, we show that the rank preservation conditions can be replaced by a group of specific rank conditions, under which the IA problem is simplified. Then, based on this technique, an iterative algorithm of IA is designed for the MIMO X channel. The algorithm is designed with limited signaling dimensions. From the simulation results, we find that the algorithm has good performances even under limited signal-to-noise ratio.**

***Index Terms*— Beamforming, iterative algorithm, MIMO systems, X channels, interference alignment.**

I. Introduction

IN RECENT years, due to the rapid development of multimedia services and the demand for ubiquitous connectivity, wireless communication has attracted a lot of interests [1], [2]. Of particular interests are (multiple-input-multiple-output) MIMO X channels (XCs) [3], which refer to communication channels with multiple transmitters and multiple receivers, where each transmitter aims to send signals to each receiver.

For the wireless channels with multiple receivers, due to the broadcast nature of the wireless transmission medium, the desired signals at each receiver can be potentially interfered by signals intended for the other receivers. Interference is one of the key factors that degrade the capacity of wireless channels. The technique *interference alignment* (IA) is developed for these channels. IA is an efficient approach to abate the effect of interferences by aligning interference signals into a space distinct from the space occupied by desired signals [4]. As such, the desired signals can be distinguished from the interference signals. IA is considered in a variety of wireless communication scenarios, including the wireless interference channels [5]–[7], broadcast (or multiple access) channels [8]–[10], cognitive radio networks [11] and MIMO X channels [3], [12], [13], to maximize the remaining interference-free dimensions for the desired signals. A review of the current status of IA is presented in [14].

In this paper, we study the IA scheme of the general MIMO XC with arbitrary numbers of transmitters and receivers, and multiple antennas at each node.

Many advanced techniques have been proposed to analyze the degrees of freedom (DoF) of MIMO interference channels and the MIMO multiway relay channels [4], [15]–[18]. Specifically, Cadambe and Jafar [4] first applied the idea of IA to analyze the DoF of interference channels, which is the benchmark for the later IA studies. The authors in [15] studied the DoF of a 3-user MIMO interference channel with a same number of antennas at each transmitter and a same number of antennas at each receiver. The DoF of the symmetric multiple input multiple output (MIMO) multipair two-way distributed relay channel has been studied in [16]. Ding *et al.* [17] studied the DoF of the symmetric multi-relay MIMO Y channel. Recently, the authors in [18] studied the DoF of the asymmetric 3-user single-relay MIMO Y channel with weighted common and private messages. The techniques proposed by these papers are under one or more following channel conditions: i) MIMO interference channel [4], [15], in which the received signals for each receiver come from the same transmitter. ii) Symmetric wireless channel [16], [17], which means that all the user nodes are with the same number of antennas, and all the relays are also with the same number of antennas. iii) With specific number of users, i.e., in [18], the number of users is assumed to be 3. Therefore, even though the research results proposed in these papers are very useful in the DoF study of the corresponding wireless channels, they still cannot be applied directly to the general MIMO X channel with arbitrary numbers of transmitters and receivers, and arbitrary numbers of antennas at each node.

The DoF for MIMO XC have been studied in [19]–[21], with closed-form expressions for transmit precoding and receive decoding matrices. The closed-form solutions are under some special conditions:

- First, some solutions are only reachable in high Signal-to-Noise-Ratio (SNR). Since the SNR in practical environments is limited, the solutions intended for high-SNR may not be applicable in real life.
- Second, the closed-form solutions can only be found for some particular cases, e.g., i) single-antenna terminals with extremely large time or frequency extension,

Manuscript received July 23, 2017; revised December 18, 2017, May 7, 2018 and November 27, 2018; accepted January 21, 2019. Date of publication February 12, 2019; date of current version March 11, 2019. This work was supported in part by the General Research Fund from the Research Grants Council, Hong Kong, under Project CUHK 14203616. The associate editor coordinating the review of this paper and approving it for publication was S. Yang. *(Corresponding author: Yi Wei.)*

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Digital Object Identifier 10.1109/TWC.2019.2897676

in which the channel matrices are with large sizes and in diagonalized form [19]; ii) 2×2 MIMO XC case [20], [21].

On general multiple-transmitter-multiple-receiver MIMO XCs with multiple antennas at each node and limited number of signaling dimensions (i.e., the channel matrices that have no special form and are with limited sizes), the closed-form solutions to the IA problem are still unknown. Actually, even for the general interference channel (IC) with multiple transmitter-receiver pairs, on which each transmitter only wants to send signals to its corresponding receiver, it is difficult to obtain the analytical solutions to the IA problems, and even the feasibility of IA over a limited number of signaling dimensions is an open problem [22]. The IA problem on XC is far more complicated than that on IC, due to the fact that the interference nulling conditions and rank preservation conditions are coupled with each other on XC.

Consider a $T \times R$ MIMO XC with T transmitters and R receivers. For each receiver, it will receive T desired signals and $TR - T$ interference signals. When the receiver is decoding one of the desired signals, all the other $T - 1$ desired signals together with the interference signals are treated as interferences and should be zero-forced by the decoder. In other words, for each desired signal, there are $TR-1$ *to-be-zero-forced signals*. The conditions for aligning the to-be-zero-forced signals into a certain subspace for each desired signal are named as *IA Conditions*. For each desired signal, we need to ensure that it lies in a subspace distinct from its corresponding to-be-zero-forced signals. Therefore, in the IA problem, some *Rank Conditions* should be added to guarantee that the desired signals can be distinguished from their to-be-zero-forced signals. On IC, the Rank Conditions are automatically satisfied [23] and thus can be removed directly. However, on XC, the Rank Conditions cannot be ignored, since the IA Conditions and Rank Conditions are coupled. This is because the desired signals for each receiver on IC are from the same transmitter, while on XC they are from different transmitters. In [15]–[17], the authors have jointly discussed the IA Conditions and the Rank Conditions in different MIMO networks. The main technique of the works are to replace the Rank Conditions with some special structures, hence to make the problem solvable. The analysis in these works is very instructive, and provides idea for our study.

Due to the complexity of finding closed-form solutions, algorithmic techniques, such as iterative IA, have been proposed to find the numerical solutions on the general IC in [22] and [24]–[26]. Some iterative IA schemes are applied to the 2×2 MIMO XC in [3] and [27], but generalizations to XC with more than 2 transmitters and 2 receivers are shown not to be straightforward.

In this paper, an iterative alternating minimization algorithm is proposed for general MIMO XC to find IA solutions. In each iteration of the algorithm, we update the receive decoding matrices or transmit precoding matrices by solving an optimization problem under the IA Conditions and the Rank Conditions. Due to the non-convex nature of the Rank Conditions, a solution to the general rank-constrained optimization problem has remained open for a long time [28]. In order to overcome the challenge of the Rank Conditions, we analyze the necessary conditions of the Rank Conditions and IA Conditions and then find that under the necessary conditions and IA Conditions, the Rank Conditions are automatically satisfied almost surely. Thus we replace the Rank Conditions with their necessary conditions, which simplifies the optimization problem. The main results of the alternating minimization algorithm introduced in this paper are as follows.

1) The algorithm is efficient for the general MIMO XC. Here the general MIMO XC refers to a channel with multiple transmitters and multiple receivers, no special form of channel matrices, limited signaling dimensions and limited SNR.
2) The algorithm can provide numerical insights into the feasibility of IA for the general MIMO XC.
3) We evaluate the performances of this algorithm and compare it with other existing algorithms. Numerical results show that the sum rate of our proposed algorithm is higher than that of comparison algorithms.

The remainder of this paper is organized as follows. Section II describes the MIMO XC. In Section III, we analyze the IA scheme for the MIMO XC and define the IA Feasible Conditions, under which perfect IA is achieved. In Section IV, an iterative IA algorithm is proposed. Performance evaluations are in Section V. Finally, Section VI concludes the paper.

Notations: All boldface letters indicate matrices (upper-case) or vectors (lower-case). $\mathbb{C}$ represents the complex domain. $\mathbf{I}_m$ denotes the $m \times m$ identity matrix and $\mathbf{0}_{m\times n}$ denotes the $m \times n$ zero matrix. $\mathbf{A}^H$ denotes the conjugate transpose of $\mathbf{A}$. $\det(\mathbf{A})$ and $\text{Tr}(\mathbf{A})$ are the determinant and trace of the matrix $\mathbf{A}$, respectively. The rank of the matrix $\mathbf{A}$ is denoted as $\text{rank}(\mathbf{A})$. $\boldsymbol{\nu}_d[\mathbf{A}]$ is the eigenvector corresponding to the d^{th} smallest eigenvalue of $\mathbf{A}$. $\mathbf{A}^{\star d}$ represents the d^{th} column of matrix $\mathbf{A}$.

II. Definition–MIMO X Channel

We consider a $T \times R$ MIMO XC. It is a communication channel with T transmitters and R receivers, where each transmitter has a distinct signal for each receiver and each node is equipped with multiple antennas. The number of antennas at each node is assumed to be M. We denote the t^{th} transmitter as T_t and denote the r^{th} receiver as R_r. $\mathcal{T} = \{1, \ldots, T\}$ and $\mathcal{R} = \{1, \ldots, R\}$ are the sets of transmitters and receivers, respectively. The global channel state information is assumed to be available at all the transmitters and receivers. An example of a MIMO XC with 3 transmitters and 2 receivers is shown in Fig. 1.

A. Transmitter Side

We assume that the t^{th} transmitter T_t wishes to transmit the signal $\mathbf{x}_{tr} \in \mathbb{C}^{D\times 1}$ to the r^{th} receiver R_r, where D represents the degree of freedom (DoF) achieved by each signal. Intuitively, D can be perceived as the number of signal space dimensions that are free of interference. We assume the average power of each component of $\mathbf{x}_{tr}$ is P_{tr}, i.e., $\mathbb{E}(\mathbf{x}_{tr}\mathbf{x}_{tr}^H) = P_{tr}\mathbf{I}_D$. We denote the channel matrix from the transmitter T_t to the receiver R_r as $\mathbf{H}_{tr} \in \mathbb{C}^{M\times M}$.

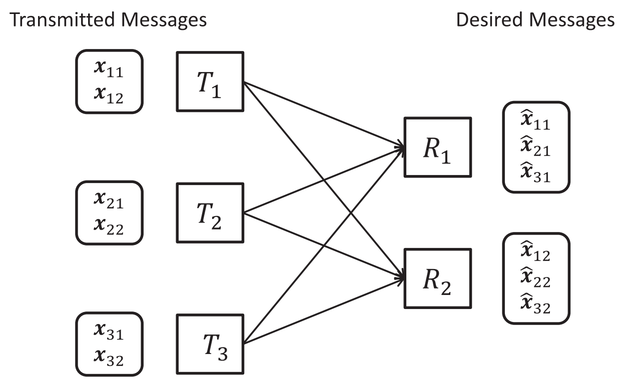

Fig. 1. A 3×2 MIMO X channel.

All the entries of the channel matrix $\mathbf{H}_{tr}$ are identically and independently distributed (i.i.d.) zero mean complex Gaussian random variables with unit variance. Thus, all channel matrices will be of full rank with probability 1.

Transmitter T_t uses linear transmit precoding matrix $\mathbf{V}_{tr} \in \mathbb{C}^{M\times D}$, with orthonormal columns, to map the D symbols in $\mathbf{x}_{tr}$ to its M antennas. On the MIMO XC, each transmitter has a distinct signal for each receiver. Thus, for the $T \times R$ MIMO XC with R receivers, the signal vector transmitted by transmitter T_t is $\sum_{r\in\mathcal{R}} \mathbf{V}_{tr}\mathbf{x}_{tr}$.

B. Receiver Side

The received signal vector for the r^{th} receiver R_r can be expressed as:

$$\mathbf{y}_r = \sum_{t\in\mathcal{T}} \mathbf{H}_{tr} \sum_{n\in\mathcal{R}} \mathbf{V}_{tn}\mathbf{x}_{tn} + \mathbf{z}_r \tag{1a}$$

$$= \sum_{t\in\mathcal{T}} \mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{x}_{tr} + \sum_{t\in\mathcal{T}} \sum_{n\in\mathcal{R}, n\neq r} \mathbf{H}_{tr}\mathbf{V}_{tn}\mathbf{x}_{tn} + \mathbf{z}_r, \tag{1b}$$

where the vector $\mathbf{z}_r$ is the unit-variance, zero-mean, complex additive white Gaussian noise.

The first part in (1b) represents receiver R_r's desired signals from all T transmitters. The second part represents the interference signals, which are signals intended for the other receivers. For each receiver, there are T desired signals and $TR - T$ interference signals.

We use $\mathbf{U}_{tr} \in \mathbb{C}^{M\times D}$ to denote the receive decoding matrix that decodes the signal transmitted from the t^{th} transmitter and desired by the r^{th} receiver (i.e., the signal $\mathbf{x}_{tr}$). The columns of $\mathbf{U}_{tr}$ are orthonormal. The received signal vector after being decoded by $\mathbf{U}_{tr}$ at receiver R_r is:

$$\bar{\mathbf{y}}_r = \mathbf{U}_{tr}^H \mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{x}_{tr} + \mathbf{U}_{tr}^H \Big(\sum_{m\in\mathcal{T}} \sum_{n\in\mathcal{R}} \mathbf{H}_{mr}\mathbf{V}_{mn}\mathbf{x}_{mn} - \mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{x}_{tr} \Big) + \mathbf{U}_{tr}^H \mathbf{z}_r.$$

Then, the *receiving rate function* of the receiver R_r corresponding to $\mathbf{x}_{tr}$ is represented by:

$$S_{tr} \triangleq \log \left(\det\Big(\mathbf{I}_D + P_{tr}\mathbf{U}_{tr}^H \mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{V}_{tr}^H \mathbf{H}_{tr}^H \mathbf{U}_{tr} \times (\mathbf{U}_{tr}^H \boldsymbol{\mathcal{I}}_{tr}\mathbf{U}_{tr} + \mathbf{I}_D)^{-1} \Big) \right), \tag{2}$$

where $\boldsymbol{\mathcal{I}}_{tr}$ is the *interference covariance matrix* corresponding to the signal $\mathbf{x}_{tr}$, and

$$\boldsymbol{\mathcal{I}}_{tr} = \sum_{m\in\mathcal{T}} \sum_{n\in\mathcal{R}} P_{mn}\mathbf{H}_{mr}\mathbf{V}_{mn}\mathbf{V}_{mn}^H \mathbf{H}_{mr}^H - P_{tr}\mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{V}_{tr}^H \mathbf{H}_{tr}^H. \tag{3}$$

If perfect IA is achieved, $\mathbf{U}_{tr}^H \boldsymbol{\mathcal{I}}_{tr}\mathbf{U}_{tr} = \mathbf{0}_{D\times D}$ for all $t \in \mathcal{T}$ and $r \in \mathcal{R}$.

In the next section, we will analyze the IA scheme of the MIMO XC, aiming to make the power of interference signals in the subspace occupied by the desired signals to be zero. In other words, we aim to let $\mathbf{U}_{tr}^H \boldsymbol{\mathcal{I}}_{tr}\mathbf{U}_{tr} = \mathbf{0}_{D\times D}$ for all $t \in \mathcal{T}$ and $r \in \mathcal{R}$.

Remark 1: For notational convenience, we made the assumption that all the nodes (transmitters and receivers) are equipped with the same number of antennas M, and each signal achieves D DoF. However, the analysis and the algorithm introduced in this paper can be used directly in the cases that the nodes are equipped with different numbers of antennas and the signals achieve different DoFs.

III. Analysis of Interference Alignment of the $T \times R$ MIMO X Channel

According to the definition of IA, for each desired signal, the interference signals should be aligned into the null-space of its decoder. Thus, when we use $\mathbf{U}_{tr}$ to decode the signals received by the receiver R_r to obtain $\mathbf{x}_{tr}$, we have the following conditions on $\mathbf{U}_{tr}$

$$\mathbf{U}_{tr}^H \mathbf{H}_{mr}\mathbf{V}_{mn} = 0, \quad \forall m \in \mathcal{T}, n \in \mathcal{R},$$

and

$$|m - t| + |n - r| \neq 0. \tag{4}$$

Conditions in (4) indicate that for the decoding matrix $\mathbf{U}_{tr}$, all the signals received by the receiver R_r (i.e., $\mathbf{H}_{mr}\mathbf{V}_{mn}\mathbf{x}_{mn}$ for all $m \in \mathcal{T}$ and $n \in \mathcal{R}$) except $\mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{x}_{tr}$ (i.e., the case that $m = t$ and $n = r$) are to-be-zero-forced signals. All these signals should be aligned into the null-space of $\mathbf{U}_{tr}$.

To perfectly decode all the desired signals for all the receivers, the conditions in (4) should be applied to all $t \in \mathcal{T}$ and $r \in \mathcal{R}$. Thus, we have the following feasible conditions, under which rank$(\mathbf{U}_{tr}^H \mathbf{H}_{tr}\mathbf{V}_{tr}) = D$ $(\forall t, r)$ and perfect IA is achieved.

*Definition 1 (IA Feasible Conditions): Given a $T \times R$ MIMO XC with M antennas at each node and with D DoF of each signal, we say that the channel is **IA feasible** if and only if there exist a set of decoding matrices $\{\mathbf{U}_{tr}\}$ and a set of precoding matrices $\{\mathbf{V}_{tr}\}$ such that the following conditions are satisfied.*

IA Conditions:

$$\mathbf{U}_{tr}^H \mathbf{H}_{mr}\mathbf{V}_{mn} = 0, \quad \forall t \in \mathcal{T},\ m \in \mathcal{T},\ r \in \mathcal{R},\ n \in \mathcal{R},$$

and

$$|m - t| + |n - r| \neq 0. \tag{5}$$

Rank Conditions:

$$\textit{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr}\mathbf{V}_{tr}) = D, \quad \forall t \in \mathcal{T},\ r \in \mathcal{R}. \tag{6}$$

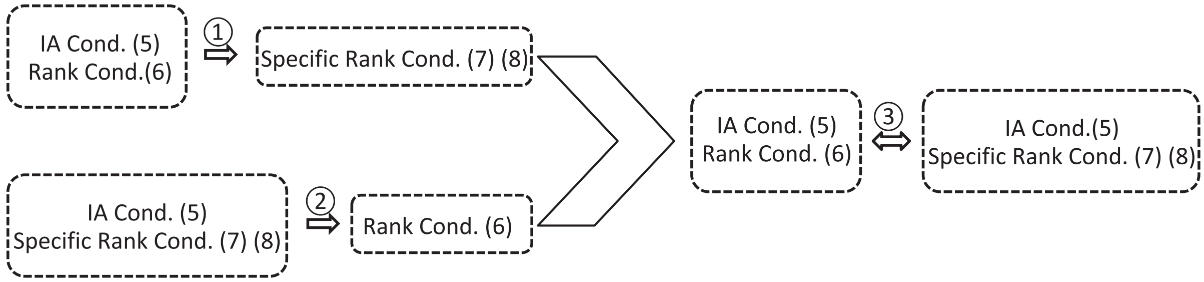

Fig. 2. Flow chart of Rank Conditions transformation (in this figure, cond. means conditions).

IA Conditions are interference nulling conditions, which are used to align the to-be-zero-forced signals into the null space of the corresponding decoding matrix. Rank Conditions are used to guarantee that each desired signal is received through a full rank matrix, so that it can be distinguished from its corresponding to-be-zero-forced signals. Note that we can consider symbol extension in time or frequency instead of multiple antennas.

Remark 2: On MIMO ICs, the Rank Conditions (i.e., $\mathbf{U}_{ii}^H\mathbf{H}_{ii}\mathbf{V}_{ii}$ should be a full rank matrix for all i) are automatically satisfied. This is because the channel matrices in Rank Conditions (i.e., $\mathbf{H}_{ii}$) do not appear in the corresponding IA Conditions (i.e., $\mathbf{U}_{ii}^H\mathbf{H}_{ji}\mathbf{V}_{jj} = 0$, $\forall i \neq j$), which means that the selection of the decoding and precoding matrices $\mathbf{U}_{ii}$ and $\mathbf{V}_{ii}$ under the IA Conditions does not depend on $\mathbf{H}_{ii}$. Since the entries of $\mathbf{H}_{ii}$ are randomly generated from a continuous distribution, the product matrix $\mathbf{U}_{ii}^H\mathbf{H}_{ii}\mathbf{V}_{ii}$ has full rank almost surely. The proof can be found in Sec. VII of [22] when the authors were explaining the reason of removing constraint (12) in their paper. However, on MIMO XCs, the Rank Conditions are not satisfied automatically.

Next, we show that the Rank Conditions of MIMO XC can be replaced by some specific rank conditions, under which the problem is easier to be solved. The logic flow is shown in Fig. 2. First, from IA Feasible Conditions (i.e., (5) and (6)), we find their necessary conditions and define them as *Specific Rank Conditions* in the following lemma.

Lemma 1: For any sets of decoding matrices $\{\mathbf{U}_{tr}\}$ and precoding matrices $\{\mathbf{V}_{tr}\}$ that satisfy the IA Feasible Conditions ((5) and (6)), the Specific Rank Conditions always hold:

$$rank(\boldsymbol{\mathcal{U}}_r) = TD, \quad \forall r \in \mathcal{R}, \tag{7}$$

$$rank(\boldsymbol{\mathcal{V}}_t) = RD, \quad \forall t \in \mathcal{T}, \tag{8}$$

where $\boldsymbol{\mathcal{U}}_r \triangleq [\mathbf{U}_{1r}, \mathbf{U}_{2r}, \ldots, \mathbf{U}_{Tr}]$ consists of all the decoding matrices at receiver R_r, and $\boldsymbol{\mathcal{V}}_t \triangleq [\mathbf{V}_{t1}, \mathbf{V}_{t2}, \ldots, \mathbf{V}_{tR}]$ consists of all the precoding matrices at transmitter T_t.

The proof of Lemma 1 is given in Appendix A.

In the proof, we define $\boldsymbol{\mathcal{U}}_{r(-t)}$ by removing the t^{th} sub-matrix $\mathbf{U}_{tr}$ from $\boldsymbol{\mathcal{U}}_r$ and define $\boldsymbol{\mathcal{V}}_{t(-r)}$ by removing the r^{th} sub-matrix $\mathbf{V}_{tr}$ from $\boldsymbol{\mathcal{V}}_t$. Specifically, we have $\boldsymbol{\mathcal{U}}_{r(-t)} \triangleq [\mathbf{U}_{1r}, \mathbf{U}_{2r}, \ldots, \mathbf{U}_{(t-1)r}, \mathbf{U}_{(t+1)r}, \ldots, \mathbf{U}_{Tr}]$ and $\boldsymbol{\mathcal{V}}_{t(-r)} \triangleq [\mathbf{V}_{t1}, \mathbf{V}_{t2}, \ldots, \mathbf{V}_{t(r-1)}, \mathbf{V}_{t(r+1)}, \ldots, \mathbf{V}_{tR}]$. These definitions will also appear in the following discussions. From Lemma 1 we know that rank$(\boldsymbol{\mathcal{U}}_r) = TD$. Since there are T sub-matrices $\{\mathbf{U}_{tr}\big| \forall t \in \mathcal{T}\}$ in $\boldsymbol{\mathcal{U}}_r$ and each of them is with rank D, the columns of $\mathbf{U}_{tr}$ are linearly independent of $\boldsymbol{\mathcal{U}}_{r(-t)}$ for all $t \in \mathcal{T}$ and $r \in \mathcal{R}$. Similarly, we can see that the columns of $\mathbf{V}_{tr}$ are linearly independent of $\boldsymbol{\mathcal{V}}_{t(-r)}$ for all $t \in \mathcal{T}$ and $r \in \mathcal{R}$.

By Lemma 1, we prove the first sign ① in Fig. 2 (i.e., (5) and (6) $\Rightarrow$ (7) and (8)). Based on the necessary conditions from Lemma 1, we have Lemma 2.

Lemma 2: For any $T \times R$ MIMO XC with M antennas at each node, if there exist a set of decoding matrices $\{\mathbf{U}_{tr}\}$ and a set of precoding matrices $\{\mathbf{V}_{tr}\}$ that satisfy IA Feasible Conditions (i.e., (5) and (6)), we have

$$D \leq \frac{M}{T+R-1}.$$

where D is the DoF achieved by each signal.

Lemma 2 follows from the outer bounds established by Cadambe and Jafar [19]. Thus, we omit the proof of Lemma 2 in our paper. Recall that D is the DoF that can be reached by each signal $\mathbf{x}_{tr}$, then the total number of DoF reached by all signals of the $T \times R$ MIMO XC is upper bounded by $\frac{TRM}{T+R-1}$.

From IA Conditions (5) and Specific Rank Conditions (7) and (8), we obtain Lemma 3.

Lemma 3: For any $T \times R$ MIMO XC with M antennas at each node, if there exist a set of decoding matrices $\{\mathbf{U}_{tr}\}$ and a set of precoding matrices $\{\mathbf{V}_{tr}\}$ that satisfy IA Conditions (5) and Specific Rank Conditions (7) and (8), then the Rank Conditions (6) are satisfied almost surely.

The proof of Lemma 3 is given in Appendix B.

As shown in Fig. 2, from the above derivation we know that IA Feasible Conditions (5) and (6) are equivalent to conditions in (5), (7) and (8). We can show that, for any set of decoding and precoding matrices that satisfy (5), (7) and (8), we can always construct matrices which also satisfy *Unitary Conditions*, i.e.,

$$\mathbf{U}_{tr}^H\mathbf{U}_{tr} = \mathbf{I}_D, \quad \mathbf{V}_{tr}^H\mathbf{V}_{tr} = \mathbf{I}_D, \ \forall t \in \mathcal{T}, \ r \in \mathcal{R}. \tag{9}$$

Hence, we call conditions in (5), (7), (8), and (9) as Rewritten Feasible Conditions.

Based on the Rewritten Feasible Conditions, in the next section, we will propose an iterative alternating minimization algorithm to find the proper decoding and precoding matrices.

IV. Iterative Interference Alignment Approach

In this section, we first introduce the IA-XC algorithm in Sec. IV-A, and then analyze its convergence property in Sec. IV-B.

A. Introduction of the IA-XC Algorithm

In this section, we aim to design an algorithm to select the proper decoding and precoding matrices, which can satisfy Rewritten Feasible Conditions (i.e., IA Conditions (5), Specific Rank Conditions (7) (8), and Unitary Conditions (9)). We can show that IA Conditions (5) can be achieved by minimizing the *interference leakage* $\mathcal{F}_i$, which is defined as:

$$\mathcal{F}_i \triangleq \sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}} \mathrm{Tr}(\mathbf{U}_{tr}^H \boldsymbol{\mathcal{L}}_{tr}\mathbf{U}_{tr}), \tag{10}$$

where

$$\boldsymbol{\mathcal{L}}_{tr} = \sum_{m\in\mathcal{T}}\sum_{n\in\mathcal{R}} \mathbf{H}_{mr}\mathbf{V}_{mn}\mathbf{V}_{mn}^H\mathbf{H}_{mr}^H - \mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{V}_{tr}^H\mathbf{H}_{tr}^H. \tag{11}$$

The interference leakage $\mathcal{F}_i$ is lower bounded by zero. The minimum value of $\mathcal{F}_i$ equals zero if perfect IA can be reached, and is larger than zero otherwise. Since $\boldsymbol{\mathcal{L}}_{tr}$ is a function of precoding matrices $\{\mathbf{V}_{tr}\}$, we write it as $\boldsymbol{\mathcal{L}}_{tr}(\mathbf{V})$. Due to the linear mapping property of trace, $\mathcal{F}_i$ can be rewritten as:

$$\begin{aligned}\mathcal{F}_i &= \mathrm{Tr}\Big(\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\mathbf{U}_{tr}^H\boldsymbol{\mathcal{L}}_{tr}(\mathbf{V})\mathbf{U}_{tr}\Big)\\ &= \mathrm{Tr}\Big(\sum_{m\in\mathcal{T}}\sum_{n\in\mathcal{R}}\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\mathbf{V}_{mn}^H\mathbf{H}_{mr}^H\mathbf{U}_{tr}\mathbf{U}_{tr}^H\mathbf{H}_{mr}\mathbf{V}_{mn}\\ &\quad -\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\mathbf{V}_{tr}^H\mathbf{H}_{tr}^H\mathbf{U}_{tr}\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tr}\Big)\\ &\overset{(a)}{=} \mathrm{Tr}\Big(\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\sum_{m\in\mathcal{T}}\sum_{n\in\mathcal{R}}\mathbf{V}_{tr}^H\mathbf{H}_{tn}^H\mathbf{U}_{mn}\mathbf{U}_{mn}^H\mathbf{H}_{tn}\mathbf{V}_{tr}\\ &\quad -\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\mathbf{V}_{tr}^H\mathbf{H}_{tr}^H\mathbf{U}_{tr}\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tr}\Big)\\ &= \sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\mathrm{Tr}\Big(\mathbf{V}_{tr}^H\tilde{\boldsymbol{\mathcal{L}}}_{tr}(\mathbf{U})\mathbf{V}_{tr}\Big),\end{aligned} \tag{12}$$

where (a) is from interchanging t with m and r with n in the first term. $\tilde{\boldsymbol{\mathcal{L}}}_{tr}(\mathbf{U})$ is defined as:

$$\tilde{\boldsymbol{\mathcal{L}}}_{tr}(\mathbf{U}) = \sum_{m\in\mathcal{T}}\sum_{n\in\mathcal{R}}\mathbf{H}_{tn}^H\mathbf{U}_{mn}\mathbf{U}_{mn}^H\mathbf{H}_{tn} - \mathbf{H}_{tr}^H\mathbf{U}_{tr}\mathbf{U}_{tr}^H\mathbf{H}_{tr}, \tag{13}$$

which is a function of decoding matrices $\{\mathbf{U}_{tr}\}$.

In order to select proper decoding and precoding matrices, which can satisfy conditions in (5), (7), (8) and (9), we formulate the minimization problem as follows:

$$\min\ \mathcal{F}_i, \tag{14a}$$
$$\text{s.t. } \mathrm{rank}(\boldsymbol{\mathcal{U}}_r) = TD,\quad \forall r\in\mathcal{R}, \tag{14b}$$
$$\mathrm{rank}(\boldsymbol{\mathcal{V}}_t) = RD,\quad \forall t\in\mathcal{T}, \tag{14c}$$
$$\mathbf{V}_{tr}^H\mathbf{V}_{tr} = \mathbf{I}_D,\quad \forall t\in\mathcal{T},\ r\in\mathcal{R}, \tag{14d}$$
$$\mathbf{U}_{tr}^H\mathbf{U}_{tr} = \mathbf{I}_D,\quad \forall t\in\mathcal{T},\ r\in\mathcal{R}. \tag{14e}$$

It is still very challenging to solve this minimization problem (14) under the rank constraints in (14b) and (14c) directly. Nevertheless, we can find a set of sufficient conditions of the constraints in (14b) and (14c) to replace them. We show the sufficient conditions in the following lemma.

Lemma 4: For any $T\times R$ MIMO XC with M antennas at each node, if there exists a set of decoding matrices $\{\mathbf{U}_{tr}\}$ and a set of precoding matrices $\{\mathbf{V}_{tr}\}$ (where $\mathbf{U}_{tr}^H\mathbf{U}_{tr}=\mathbf{I}_D$ and $\mathbf{V}_{tr}^H\mathbf{V}_{tr}^H=\mathbf{I}_D$) that satisfy

$$\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{T},m>t} Tr(\mathbf{U}_{tr}^H\mathbf{U}_{mr}\mathbf{U}_{mr}^H\mathbf{U}_{tr}) < \frac{T}{2(T-1)},\quad \forall r\in\mathcal{R}, \tag{15}$$

$$\sum_{r\in\mathcal{R}}\sum_{n\in\mathcal{R},n>r} Tr(\mathbf{V}_{tr}^H\mathbf{V}_{tn}\mathbf{V}_{tn}^H\mathbf{V}_{tr}) < \frac{R}{2(R-1)},\quad \forall t\in\mathcal{T} \tag{16}$$

then the constraints in (14b) *and* (14c) *are satisfied.*

The proof of Lemma 4 is given in Appendix C.

Based on Lemma 4, we can solve the original optimization problem (14) by solving its inner approximation problem[1]

$$\begin{aligned}&\min\ \mathcal{F}_i,\\ &\text{s.t. conditions in (15) and (16)},\\ &\quad \mathbf{V}_{tr}^H\mathbf{V}_{tr} = \mathbf{I}_D,\quad \forall t\in\mathcal{T},\ r\in\mathcal{R},\\ &\quad \mathbf{U}_{tr}^H\mathbf{U}_{tr} = \mathbf{I}_D,\quad \forall t\in\mathcal{T},\ r\in\mathcal{R}.\end{aligned} \tag{17}$$

The main difficulty in solving the nonlinear nonconvex optimization problem (17) is due to the nonlinear inequality constraints (15) and (16). Motivated by Courant penalty function technique,[2] in our algorithm, we scale each condition in (15) and (16) by a penalty parameter $\omega_u^{[r]}$ (or $\omega_v^{[t]}$), and move them to the objective function. Specifically, we apply penalty technique to the inequality constraints and keep the unitary constraints in order to obtain the following problem:

$$\begin{aligned}&\min\ \mathcal{F} \triangleq \mathcal{F}_i + \sum_{r\in\mathcal{R}}\omega_u^{[r]}\mathcal{F}_u^{[r]} + \sum_{t\in\mathcal{T}}\omega_v^{[t]}\mathcal{F}_v^{[t]},\\ &\text{s.t.} \begin{cases}\mathbf{V}_{tr}^H\mathbf{V}_{tr} = \mathbf{I}_D,\quad \forall t\in\mathcal{T},\ r\in\mathcal{R},\\ \mathbf{U}_{tr}^H\mathbf{U}_{tr} = \mathbf{I}_D,\quad \forall t\in\mathcal{T},\ r\in\mathcal{R}.\end{cases}\end{aligned} \tag{18}$$

where

$$\mathcal{F}_u^{[r]} = \sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{T},m>t}\mathrm{Tr}(\mathbf{U}_{tr}^H\mathbf{U}_{mr}\mathbf{U}_{mr}^H\mathbf{U}_{tr}), \tag{19}$$

and

$$\mathcal{F}_v^{[t]} = \sum_{r\in\mathcal{R}}\sum_{n\in\mathcal{R},n>r}\mathrm{Tr}\Big(\mathbf{V}_{tr}^H\mathbf{V}_{tn}\mathbf{V}_{tn}^H\mathbf{V}_{tr}\Big). \tag{20}$$

Based on [30] and [31], the solution of (17) can be approximated by that of (18) as the penalty parameters approach zero. In (18), larger penalty parameters (i.e., $\omega_u^{[r]}$ $\omega_v^{[t]}$) lead to

[1]Since problem (17) is the inner approximation problem of the original optimization problem (14), the minimum interference leakage $\mathcal{F}_i$ achieved by solving (17) may be a sub-optimal value of the problem (14). In other words, the algorithm, which is designed to solve the problem (17), can only give the lower bound of the DoF that can be achieved by general MIMO XCs.

[2]Courant penalty function technique is a classic penalty function technique, which avoids dealing with constraints by moving them to the objective function [29].

smaller value of $\mathcal{F}_u^{[r]}$ and $\mathcal{F}_v^{[t]}$, but they may also lead to a larger value of $\mathcal{F}_i$ (i.e., the real objective function). To avoid the negative influence of large penalty parameters and to preserve the positive influence, we initialize our algorithm with large positive $\omega_u^{[r]}$ and $\omega_v^{[t]}$, which refer to more strict conditions than (15) and (16).

In our algorithm, there are two layers of iterations. In the inner layer iteration, we fix the penalty parameters and iteratively minimize $\mathcal{F}$ (see Eq. (18)). We divide the variables into two groups, where the first group contains all the precoding matrices $\{\mathbf{V}_{tr}\}$, and the other group contains all the decoding matrices $\{\mathbf{U}_{tr}\}$. We initialize the algorithm by randomly generating TR precoding matrices $\mathbf{V}_{tr}$ and TR decoding matrices $\mathbf{U}_{tr}$, and iterate the following steps. In the first step, we keep all precoding matrices unchanged and minimize $\mathcal{F}$ by tuning decoding matrices. In the second step, we keep decoding matrices (which are obtained by solving the problem in the previous step) unchanged and update precoding matrices to minimize the objective function $\mathcal{F}$. If sufficient reduction on the interference leakage $\mathcal{F}_i$ in the inner layer iteration is achieved, we stop our algorithm and output the decoding and precoding matrices. Otherwise, we continue the outer layer iteration, in which we update the penalty parameters $\omega_u^{[r]}$ with a smaller value $h_u(\mathcal{F}_u^{[r]})\omega_u^{[r]}$ and update $\omega_v^{[t]}$ with $h_v(\mathcal{F}_v^{[t]})\omega_v^{[t]}$. The *decreasing rate functions* $h_u(x)$ and $h_v(x)$ are in $[0,1]$ and increasing in x. When we replace the original penalty parameters $\omega_u^{[r]}$ and $\omega_v^{[t]}$ with the updated ones $h_u(\mathcal{F}_u^{[r]})\omega_u^{[r]}$ and $h_v(\mathcal{F}_v^{[t]})\omega_v^{[t]}$, the solutions of (18) with the original penalty parameters provide good initial points for the next round of iterations with the updated penalty parameters. Specifically, the algorithm contains the following steps.

- **Step I**: With fixed precoding matrices $\{\mathbf{V}_{tr}\}$, we update decoding matrices $\{\mathbf{U}_{tr}\}$ in order to minimize $\mathcal{F}$. Respect to $\{\mathbf{U}_{tr}\}$, we have:

$$\mathcal{F}_{[\mathbf{U}]} = \sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}} \mathrm{Tr}\big(\mathbf{U}_{tr}^H \boldsymbol{\mathcal{L}}_{tr}(\mathbf{V})\mathbf{U}_{tr}\big) + \sum_{r\in\mathcal{R}}\omega_u^{[r]}\sum_{t\in\mathcal{T}}\sum_{m\in\mathcal{T},m>t}\mathrm{Tr}(\mathbf{U}_{tr}^H\mathbf{U}_{mr}\mathbf{U}_{mr}^H\mathbf{U}_{tr}). \tag{21}$$

By defining

$$\mathbf{Q}_{tr} = \boldsymbol{\mathcal{L}}_{tr}(\mathbf{V}) + \omega_u^{[r]}\sum_{m\in\mathcal{T},m\neq t}\mathbf{U}_{mr}\mathbf{U}_{mr}^H, \tag{22}$$

we can find that the formula $\mathrm{Tr}(\mathbf{U}_{tr}^H\mathbf{Q}_{tr}\mathbf{U}_{tr})$ contains all the terms, which include $\mathbf{U}_{tr}$ in $\mathcal{F}$. Thus for fixed precoding matrices $\{\mathbf{V}_{tr}\}$ and decoding matrices $\{\mathbf{U}_{mn} \mid |m-t|+|n-r|\neq 0\}$, we can minimize $\mathcal{F}$ respect to $\mathbf{U}_{tr}$ by minimizing $\mathrm{Tr}(\mathbf{U}_{tr}^H\mathbf{Q}_{tr}\mathbf{U}_{tr})$.
In order to minimize $\mathrm{Tr}(\mathbf{U}_{tr}^H\mathbf{Q}_{tr}\mathbf{U}_{tr})$, the columns of $\mathbf{U}_{tr}$ should be the eigenvectors corresponding to the D smallest eigenvalues of $\mathbf{Q}_{tr}$. Then, the d^{th} column of $\mathbf{U}_{tr}$ is:

$$\mathbf{U}_{tr}^{\star d} = \boldsymbol{\nu}_d(\mathbf{Q}_{tr}). \tag{23}$$

If decoding matrices $\{\mathbf{U}_{tr}\}$ obtained in this step satisfy the constraints in (15), we update them and move to Step II. Otherwise, we stop the algorithm and output $\{\mathbf{U}_{tr}\}$ and $\{\mathbf{V}_{tr}\}$, which are from the previous recursion.

- **Step II**: With fixed decoding matrices obtained from Step I, the goal of this step is to find precoding matrices $\{\mathbf{V}_{tr}\}$ to minimize $\mathcal{F}$. Respect to $\{\mathbf{V}_{tr}\}$, we have

$$\mathcal{F}_{[\mathbf{V}]} = \sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}} \mathrm{Tr}\big(\mathbf{V}_{tr}^H \tilde{\boldsymbol{\mathcal{L}}}_{tr}(\mathbf{U})\mathbf{V}_{tr}\big) + \sum_{t\in\mathcal{T}}\omega_v^{[t]}\sum_{r\in\mathcal{R}}\sum_{n\in\mathcal{R},n>r}\mathrm{Tr}(\mathbf{V}_{tr}^H\mathbf{V}_{tn}\mathbf{V}_{tn}^H\mathbf{V}_{tr}). \tag{24}$$

By defining

$$\tilde{\mathbf{Q}}_{tr} = \tilde{\boldsymbol{\mathcal{L}}}_{tr}(\mathbf{U}) + \omega_v^{[t]}\sum_{n\in\mathcal{R},n\neq r}\mathbf{V}_{tn}\mathbf{V}_{tn}^H, \tag{25}$$

we can find that the formula $\mathrm{Tr}(\mathbf{V}_{tr}^H\tilde{\mathbf{Q}}_{tr}\mathbf{V}_{tr})$ contains all the terms, which include $\mathbf{V}_{tr}$ in $\mathcal{F}$. Hence, for $\mathbf{V}_{tr}$, we can minimize $\mathcal{F}$ by minimizing $\mathrm{Tr}(\mathbf{V}_{tr}^H\tilde{\mathbf{Q}}_{tr}\mathbf{V}_{tr})$. Same as in Step I, the d^{th} column of $\mathbf{V}_{tr}$ is given by $\mathbf{V}_{tr}^{\star d} = \boldsymbol{\nu}_d(\tilde{\mathbf{Q}}_{tr})$. If the new precoding matrices $\{\mathbf{V}_{tr}\}$ satisfy the constraints in (16), we update them and move to Step I. Otherwise, we stop the algorithm and output $\{\mathbf{U}_{tr}\}$ and $\{\mathbf{V}_{tr}\}$, which are from the previous recursion. In the algorithm, we alternatively update $\{\mathbf{U}_{tr}\}$ and $\{\mathbf{V}_{tr}\}$ by repeating Step I and Step II until convergence.
- **Step III (Adjust penalty parameters):** After the convergence is reached, if sufficient reduction in $\mathcal{F}_i$ is achieved, we stop the algorithm and output $\{\mathbf{V}_{tr}\}$ and $\{\mathbf{U}_{tr}\}$. Otherwise, we update the penalty parameters and move to Step I. Specifically, we replace $\omega_u^{[r]}$ with $h_u(\mathcal{F}_u^{[r]})\omega_u^{[r]}$ and replace $\omega_v^{[t]}$ with $h_v(\mathcal{F}_v^{[t]})\omega_v^{[t]}$, where $h_u(\mathcal{F}_u^{[r]})$ and $h_v(\mathcal{F}_v^{[t]})$ should have the following properties:
  1) The values of both functions are less than 1 in their feasible sets. Under this assumption, the penalty parameters keep deceasing in each outer iteration. Therefore, the convergence of the algorithm is guaranteed.
  2) Both $h_u(\mathcal{F}_u^{[r]})$ and $h_v(\mathcal{F}_v^{[t]})$ are increasing in $\mathcal{F}_u^{[r]}$ and $\mathcal{F}_v^{[t]}$, respectively. Moreover, the values of $h_u(\mathcal{F}_u^{[r]})$ and $h_v(\mathcal{F}_v^{[t]})$ are close to 1 when $\mathcal{F}_u^{[r]}$ and $\mathcal{F}_v^{[t]}$ approach the upper bound of their feasible set, respectively. We make this assumption in order to guarantee that the inequality constraints (15) and (16) are always satisfied.

  In this paper, we simply define $h_u(\mathcal{F}_u^{[r]})$ and $h_v(\mathcal{F}_v^{[t]})$ as follows:

$$h_u(\mathcal{F}_u^{[r]}) = C + \frac{(1-C)\mathcal{F}_u^{[r]}}{\frac{T}{2(T-1)}}, \quad h_v(\mathcal{F}_v^{[t]}) = C + \frac{(1-C)\mathcal{F}_v^{[t]}}{\frac{R}{2(R-1)}}, \tag{26}$$

  where $C\in[0,1]$ is the minimum value of the decreasing rate functions.

The details of the algorithm are illustrated in Algorithm 1. In the simulation, we assume that perfect IA is achieved if $\mathcal{F}_i$ is smaller than a small positive value ϵ_2, where $\epsilon_2 = 10^{-5}$.

Algorithm 1: Iterative IA-XC Algorithm

Initialize TR arbitrary precoding matrices $\{\mathbf{V}_{tr}\}$ and TR arbitrary decoding matrices $\{\mathbf{U}_{tr}\}$. $\mathbf{V}_{tr} \in \mathbb{C}^{M\times D}$, $\mathbf{V}_{tr}^H\mathbf{V}_{tr} = \mathbf{I}_D$; $\mathbf{U}_{tr} \in \mathbb{C}^{M\times D}$, $\mathbf{U}_{tr}^H\mathbf{U}_{tr} = \mathbf{I}_D$.
Set the penalty parameters $\omega_u^{[r]}$ and $\omega_v^{[t]}$ to a relatively large value.
Calculate $\mathcal{F}_i$ (see (12)) and set $\bar{\mathcal{F}}_i = \mathcal{F}_i$.
Let $\bar{\mathbf{V}}_{tr} = \mathbf{V}_{tr}$ and $\bar{\mathbf{U}}_{tr} = \mathbf{U}_{tr}$.
repeat
 Step I:
 From the 1^{st} receiver to the R^{th} receiver.
 for $t = 1$; $t \leq T$; $t++$ **do**
  Calculate $\boldsymbol{\mathcal{L}}_{tr}(\mathbf{V})$ and $\mathbf{Q}_{tr}$ as defined in (11) and (22), respectively.
  Update the decoding matrix $\mathbf{U}_{tr}$, $\mathbf{U}_{tr}^{\star d} = \boldsymbol{\nu}_d(\mathbf{Q}_{tr})$.
  if $\mathbf{U}_{tr}$ *does not satisfy* (15) **then**
   Stop the algorithm and output $\{\bar{\mathbf{U}}_{tr}\}$ $\{\bar{\mathbf{V}}_{tr}\}$.
  else
   Set $\bar{\mathbf{V}}_{tr} = \mathbf{V}_{tr}$ and $\bar{\mathbf{U}}_{tr} = \mathbf{U}_{tr}$.
 Step II:
 From the 1^{st} transmitter to the T^{th} transmitter.
 for $r = 1$; $r \leq R$; $r++$ **do**
  Calculate $\tilde{\boldsymbol{\mathcal{L}}}_{tr}(\mathbf{U})$ and $\tilde{\mathbf{Q}}_{tr}$ as defined in (13) and (25), respectively.
  Update the precoding matrix $\mathbf{V}_{tr}$, $\mathbf{V}_{tr}^{\star d} = \boldsymbol{\nu}_d(\tilde{\mathbf{Q}}_{tr})$.
  if $\mathbf{V}_{tr}$ *does not satisfy* (16) **then**
   Stop the algorithm and output $\{\bar{\mathbf{U}}_{tr}\}$ $\{\bar{\mathbf{V}}_{tr}\}$.
  else
   Set $\bar{\mathbf{V}}_{tr} = \mathbf{V}_{tr}$ and $\bar{\mathbf{U}}_{tr} = \mathbf{U}_{tr}$.
until *convergence*;
Step III:
Calculate the value of $\mathcal{F}_i$.
if $\mathcal{F}_i < \epsilon_2$ **then**
 Stop the algorithm and output $\{\mathbf{V}_{tr}\}$, $\{\mathbf{U}_{tr}\}$.
else
 Decrease $\omega_u^{[r]}$ to $h_u(\mathcal{F}_u^{[r]})\omega_u^{[r]}$ and decrease $\omega_v^{[t]}$ to $h_v(\mathcal{F}_v^{[t]})\omega_v^{[t]}$. Set $\bar{\mathcal{F}}_i = \mathcal{F}_i$, $\bar{\mathbf{V}}_{tr} = \mathbf{V}_{tr}$ and $\bar{\mathbf{U}}_{tr} = \mathbf{U}_{tr}$.
 Go back to Step I

B. Convergence and Analysis of the IA-XC Algorithm

1) Convergence of the Algorithm: In IA-XC algorithm, with arbitrary initialized matrices, we alternatively update the decoding and precoding matrices in order to minimize the objective function $\mathcal{F}$. The objective function $\mathcal{F}$ is lower bounded by zero since it is the summation of several traces of positive semi-definite matrices.

In Step I and Step II of IA-XC algorithm, we try to adjust decoding or precoding matrices to minimize $\mathcal{F}$. In Step III, we decrease the value of the penalty parameters if sufficient reduction on the interference leakage $\mathcal{F}_i$ is not achieved. Since $\mathcal{F} = \mathcal{F}_i + \sum_{r\in\mathcal{R}}\omega_u^{[r]}\mathcal{F}_u^{[r]} + \sum_{t\in\mathcal{T}}\omega_v^{[t]}\mathcal{F}_v^{[t]}$ and $\mathcal{F}_u^{[r]}$ and $\mathcal{F}_v^{[t]}$ are always positive, reducing the penalty parameters is equivalent to reducing $\mathcal{F}$. Therefore, the value of $\mathcal{F}$ is monotonically reduced to zero or a positive value, which means that the algorithm will converge.

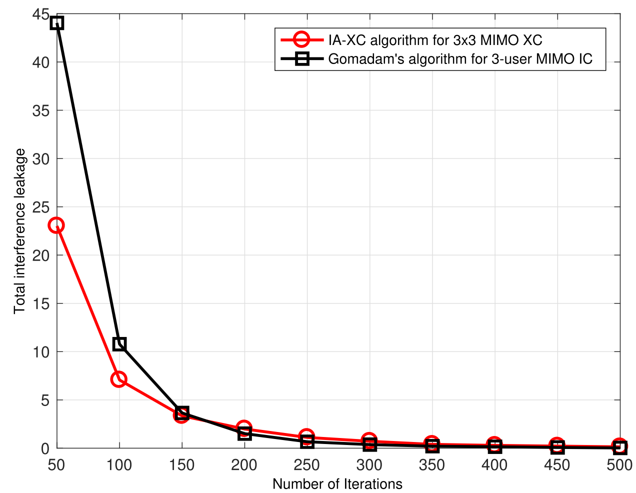

Fig. 3. Convergence example of our IA-XC algorithm with $T = R = 3$, $M = 6$.

Note that for a given MIMO X channel, the algorithm will converge for any DoF requirement of each signal (i.e., for any D) or any number of antennas equipped at each node (i.e., for any M). However, for the case that D is too large or M is too small, the value of $\mathcal{F}_i$ will converge to some positive value, in which the IA is not achieved. The examples that the values of M are too small and $\mathcal{F}_i$ converge to some positive values are shown in Fig. 3 in the simulation. Besides, in the simulation, we will give numerical insights of the feasible region of D or M for some MIMO XCs that can achieve IA by our IA-XC algorithm.

2) Convergence Rate of the Algorithm: We present an example to show the convergence rate. We compare the convergence rate of our IA-XC algorithm with a classical iterative algorithm for interference channels, which is proposed by Gomadam *et al.* [22], by plotting the convergence curves of the total interference leakage versus the number of iterations in Fig. 3. In the comparison we use a 3×3 MIMO wireless channel as an example. The number of antennas at each node is set to be 6. The curves in the figure show that our IA-XC algorithm's convergence rate is similar to the Gomadam's algorithm's convergence rate. Therefore, we can claim that our IA-XC algorithm has an acceptable convergence rate.

3) Analysis of the Algorithm: In IA-XC algorithm, if $\mathcal{F}_i$ converges to zero, IA Conditions are satisfied. In this case, perfect IA can be achieved on this $T \times R$ MIMO XC with M antennas at each node and D DoF of each signal. The sum rate of the channel is then

$$\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}}\log\left(\det(\mathbf{I}_D + P_{tr}\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tr}\mathbf{V}_{tr}^H\mathbf{H}_{tr}^H\mathbf{U}_{tr})\right). \quad (27)$$

On the contrary, if $\mathcal{F}_i$ converges to some positive value, the interference signals are not perfectly aligned and the sum rate of the MIMO XC should be written as $\sum_{t\in\mathcal{T}}\sum_{r\in\mathcal{R}} S_{tr}$, where S_{tr} defined in (2) is the receiving rate function corresponding to the signal $\mathbf{x}_{tr}$.

Actually, given a certain MIMO XC, it is not guaranteed that we can find a set of decoding and precoding matrices that satisfy Rewritten Feasible Conditions. The MIMO XC has no

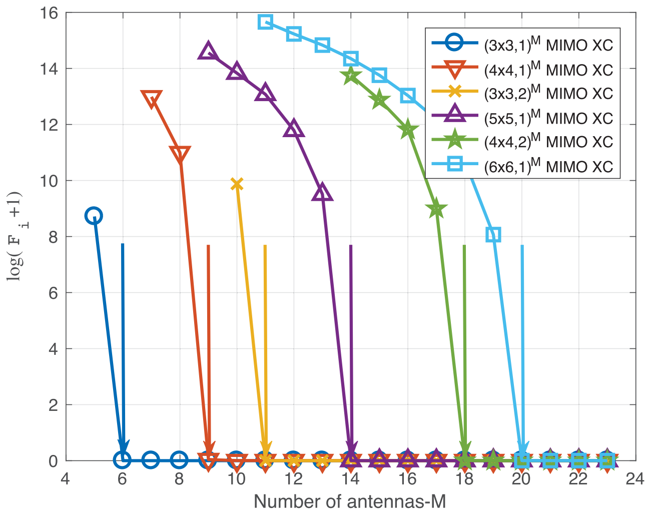

Fig. 4. The changes of $\mathcal{F}_i$ of IA-XC algorithm respect to the number of antennas at each node.

closed-form solution in general. In this paper, we have shown an iterative IA-XC algorithm, which is helpful in obtaining numerical insights into this problem.

V. Performance Evaluations

In this paper, the main goal is to provide an iterative algorithm to achieve IA of general MIMO XCs. Therefore, for different MIMO XCs, we show the DoF achieved by our IA-XC algorithm in Sec. V-A. At the meantime, we are also interested in the performance of our proposed algorithm in terms of transmission rate. Hence, we first compare the transmission rate of our IA-XC algorithm with that of the existing IA algorithms in Sec. V-B. Then, we evaluate the performance of our IA-XC algorithm in asymmetric system in Sec. V-C.

Since the IA-XC algorithm is designed to perfectly align the interference signals, it can be applied directly to any power allocation. In the simulation, we assume that the transmission power is equally allocated at each transmitter for simplicity of presentation. Specifically, we have $P_{tr} = P_{mn}$ for all t, r, m and n. In this paper, we model the entries of channel matrices $\mathbf{H}_{tr}$ as identically and independently distributed (i.i.d.) Gaussian random variables with zero mean. We initialize the penalty parameters $\omega_u^{[r]}$ and $\omega_v^{[t]}$ as $(T+R)$.

A. Feasible-Antenna-Numbers by IA-XC Algorithm

To simplify the notation, we use $(T \times R, D)^M$ to denote the MIMO XC with T transmitters and R receivers, where each transmitter and receiver has M antennas, and each signal demands D DoF.

In this subsection, we study the number of antennas needed to achieve IA on several MIMO XCs with different numbers of transmitters and receivers. With constant DoF for each signal, Fig. 4 shows the changes of the value of $\mathcal{F}_i$ in log form (i.e., $log(10^5\mathcal{F}_i + 1)$) with respect to the number of antennas at each node. According to Lemma 2, we know that D is upper-bounded by $\frac{M}{T+R-1}$, or M is lower-bounded by $D(T+R-1)$. As such, we increase the number of antennas at each node (i.e., M) from $D(T+R-1)$ for each XC. Fig. 4 shows that the objective function $\mathcal{F}_i$ decreases with the number of antennas. We call the number of antennas that can lead to perfect IA on different MIMO XCs as *feasible-antenna-numbers*, and the smallest feasible-antenna-numbers on each XC are pointed by the corresponding arrows. For example, as shown in Fig. 4, for the $(3\times3,1)^M$ MIMO XC, the feasible-antenna-numbers M are $\{6, 7, \ldots\}$. The smallest feasible-antenna-numbers M for $(3\times3,1)^M$, $(4\times4,1)^M$, $(5\times5,1)^M$, $(6\times6,1)^M$, $(3\times3,2)^M$ and $(4\times4,2)^M$ MIMO XCs are 6, 9, 14, 20, 11 and 18, respectively. Since they are larger than $D(T+R-1)$, Lemma 2 is verified. In addition, the gap between $D(T+R-1)$ and the smallest feasible-antenna-number increases with the size of XC.

B. Comparison Between IA-XC Algorithm and the Existing IA Algorithms

In this section, we will compare our proposed IA-XC algorithm with two existing IA algorithms in terms of transmission rate. To our best knowledge, IA scheme for the general MIMO XC with finite signaling dimensions is rarely studied. As such, we will compare our IA-XC algorithm with two iterative IA algorithms, which are designed for MIMO broadcasting channels (BCs) and MIMO ICs, respectively. Since these three IA algorithms are applied on different wireless channels, they have totally different IA Conditions and Rank Conditions, and the ideas behind these three algorithms are significantly different. In the rest of our paper, we call the existing IA algorithm designed for BCs as *IA-BC algorithm*, and call the existing IA algorithm designed for ICs as *IA-IC algorithm*.

First, we show the comparison between our IA-XC algorithm and the IA-BC algorithm [32]. BC is a kind of channel with a single transmitter and multiple receivers. In each time slot, the transmitter transmits distinct signals to each receiver simultaneously. Therefore, the IA-BC algorithm can only be applied on $1 \times R$ wireless channels. We assume that there are T BCs with the same R receivers, and let each transmitter broadcast its signals for one time slot. As shown in our example in Fig. 5(b), after T times slots, each transmitter has transmitted distinct signals to each receiver. To guarantee the fairness of the comparison, in the IA-BC algorithm, we scaled the transmission power for each component of signal $\mathbf{x}_{tr}$ to TP_{tr}.

We compare the sum rate of our IA-XC algorithm applied on 3×3 MIMO XCs with that of IA-BC algorithm applied on 3 3-receiver BCs in Fig. 6. When the number of antennas at each node is set to be 6, the total DoF achieved by IA-XC algorithm and IA-BC algorithm are 9 and 6, respectively. If we increase the number of antennas to 11, the total DoF achieved by these two algorithms become 18 and 11. We can see that under our simulation settings, the IA-XC algorithm can reach higher sum rate than IA-BC algorithm in high SNR region. When the transmission power is larger than 40dB, the sum rate of our proposed IA-XC algorithm is over 25% higher than that of the IA-BC algorithm in [32]. This is because that the IA-XC algorithm achieves higher total DoF comparing

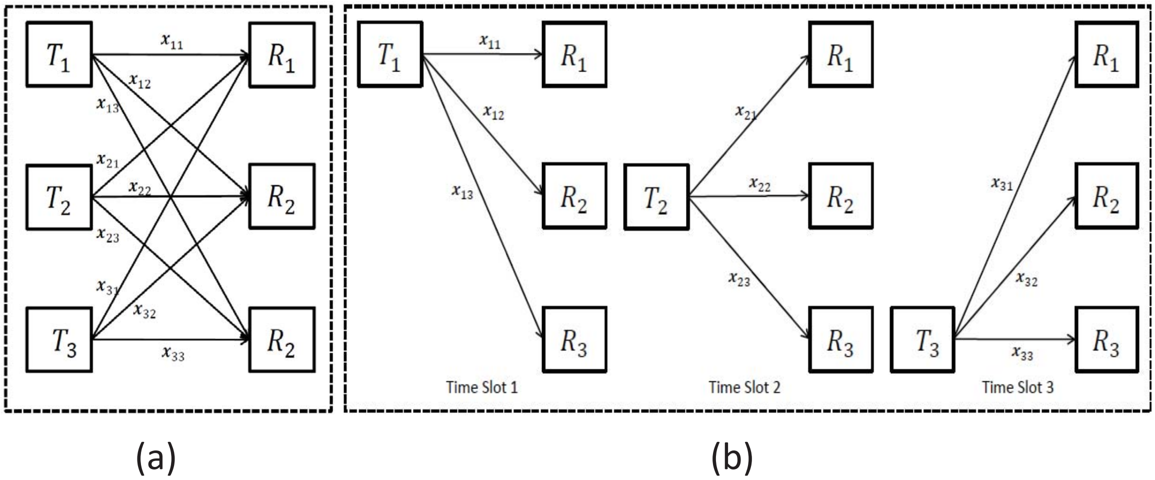

Fig. 5. An example of the systems of the comparison algorithms. (a) 3×3 MIMO XC. (b) 3 3-user MIMO BCs.

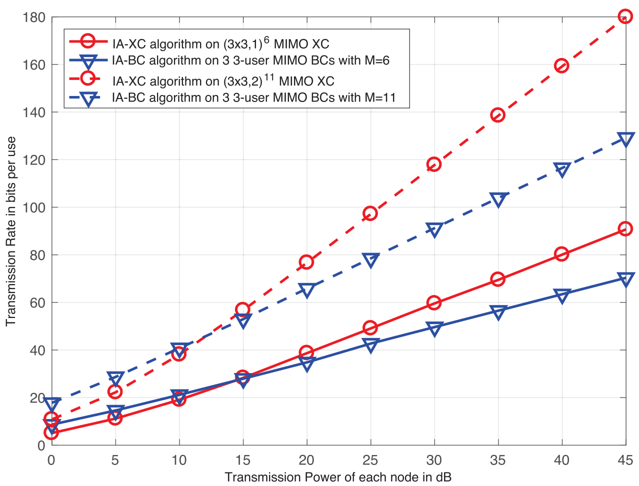

Fig. 6. Comparison between IA-XC algorithm on 3×3 MIMO XC and IA-BC algorithm on 3 3-receiver BCs.

with the IA-BC algorithm, and higher DoF guarantees higher rate in the high SNR region. However, higher DoF does not guarantee higher rate in the low SNR region. In low SNR region, the higher data rate might be achieved by some other beamforming techniques. From Fig. 6, we can find that although IA-XC algorithm has higher DoF than the IA-BC algorithm, it achieves lower sum rate than IA-BC algorithm in low SNR region. This is because our proposed IA-XC algorithm only focuses on aligning the interference signals rather than maximizing the data rate, while the IA-BC algorithm aligns interference signals and increases the data rate simultaneously.

Second, we use the iterative IA algorithm in [22], which is designed for MIMO ICs, as a benchmark and compare its performances with our IA-XC algorithm's performance. IC is a kind of channel with multiple transmitter-receiver pairs, where each transmitter only wants to send signals to its corresponding receiver. Hence, we can treat a $T \times R$ MIMO XC as $\max\{T, R\}$ $\min\{T, R\}$-user MIMO ICs and let these $\min\{T, R\}$-user MIMO ICs sequentially transmit their signals, where each IC can occupy one time slot. As a result, after $\max\{T, R\}$ time slots, each transmitter has transmitted distinct signals to each receiver. The example is shown in Fig. 7. The comparison between the two algorithms are divided into two cases: i) the wireless channel with different numbers of transmitters and receivers, i.e., $T \neq R$; ii) the wireless channel with the same numbers of transmitters and receivers, i.e., $T = R$.

As shown in Fig. 7(b), when $T \neq R$, there will be an under-utilization of transmitters (or receivers) on each time slot of MIMO ICs. Therefore, on the wireless channels with different numbers of transmitters and receivers, there is a significant advantage of MIMO XC, due to the fact that each transmitter has distinct signals to each receiver and all these nodes can be fully utilized. In Fig. 8, two 3×4 wireless channels with different antenna numbers M are studied. When $M = 7$, the total DoF achieved by IA-XC algorithm and IA-IC algorithm are 12 and 10, respectively. If we increase the antenna number to 14, the total DoF achieved by IA-XC and IA-IC algorithms are 24 and 21, respectively. We can see that under our simulation settings, the sum rate of our proposed IA-XC algorithm is always higher than that of the comparison algorithm. Moreover, when the transmission power is larger than 40dB, the increment is over 30%.

For the wireless channel with the same numbers of transmitters and receivers, i.e., $T = R$, the comparison between our IA-XC algorithm and IA-IC algorithm is illustrated. In Fig. 9, two 3×3 wireless channels with different numbers of antennas M are studied. We first set $M = 6$. In this case, the total DoF achieved by IA-XC and IA-IC algorithms are both 9. Then, we increase M to be 11. In this case, the total DoF achieved by these two algorithms are 18 and 16, respectively. From the figure we can find that the transmission rate achieved by IA-XC algorithm is slightly higher than the comparison algorithm. In conclusion, our IA-XC algorithm has higher transmission rate than IA-IC algorithm for both $T \neq R$ and $T = R$ cases.

C. Performance of IA-XC Algorithm in Asymmetric System

In this paper, we assume that the MIMO XCs are with the same number of antennas at each node (i.e., M) and the same DoF of each signal (i.e., D). In other words, we introduce our algorithm based on symmetric MIMO XCs. However, our proposed IA-XC algorithm is not limited to symmetric MIMO

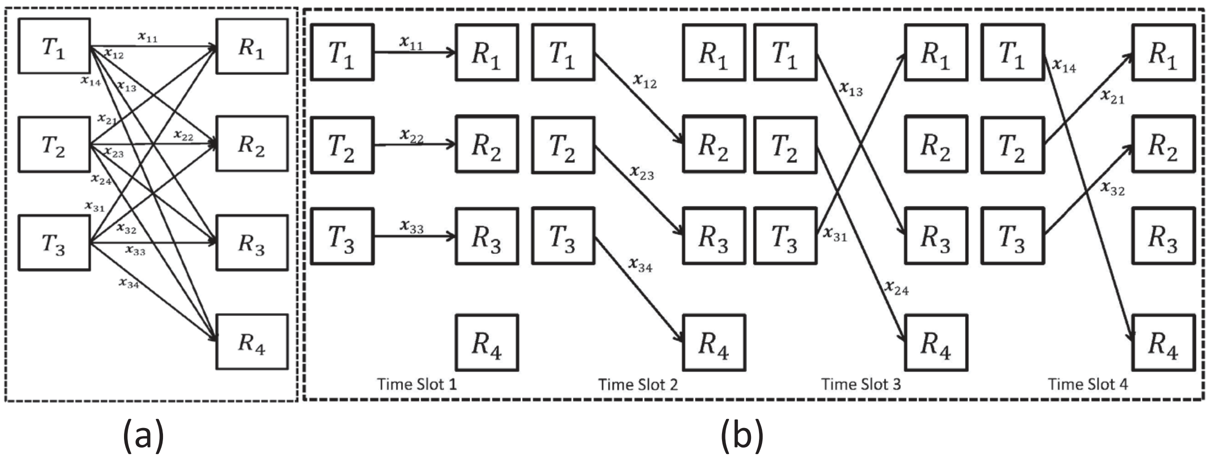

Fig. 7. An example of the systems of the comparison algorithms. (a) 3×4 MIMO XC. (b) 4 3-user MIMO ICs.

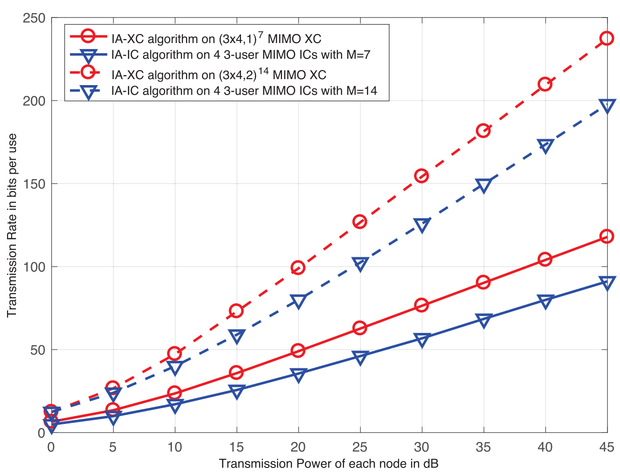

Fig. 8. Comparison between IA-XC algorithm on 3×4 MIMO XC and IA-IC algorithm on 4 3-user ICs.

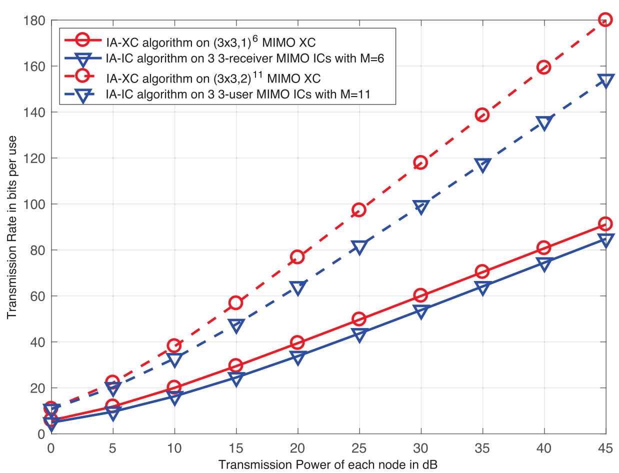

Fig. 9. Comparison between IA-XC algorithm on 3×3 MIMO XC and IA-IC algorithm on 3 3-user ICs.

XCs, but also available on asymmetric MIMO XCs, which are with different antennas at each node and are with different DoFs achieved by each signal. In asymmetric MIMO XCs, we use $M_T^{[t]}$ and $M_R^{[r]}$ to denote the numbers of antennas at the t^{th} transmitter and the r^{th} receiver, respectively. The DoF achieved by the signal transmitted from the t^{th} transmitter to the r^{th} receiver (i.e., the signal $\mathbf{x}_{tr}$) is denoted as D_{tr}.

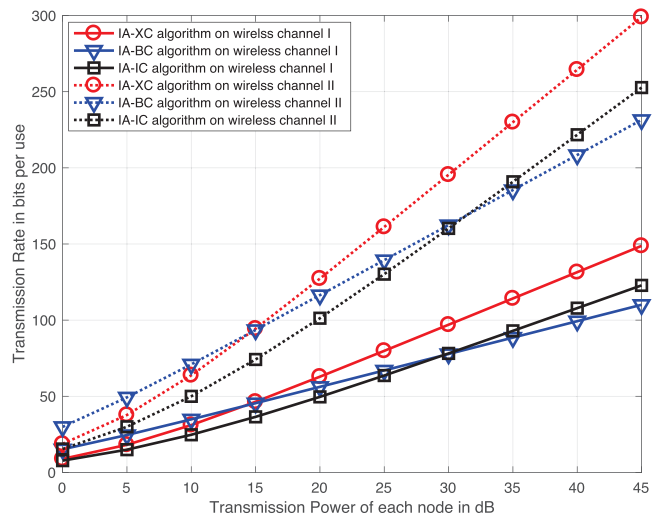

Fig. 10. Comparison between IA-XC algorithm and existing IA algorithms on 3×3 asymmetric wireless channel.

From the simulation result, we know that for the asymmetric 3×3 *wireless channel I* with $M_R^{[1]} = 6$ and $M_T^{[1]} = M_T^{[2]} = M_T^{[3]} = M_R^{[2]} = M_R^{[3]} = 10$, the DoF that can be achieved by IA-XC algorithm is $\{D_{tr}\}$, where $D_{tr} = 1\ \forall r = 1$ and $D_{tr} = 2\ \forall r \neq 1$. For the asymmetric 3×3 *wireless channel II* with $M_R^{[1]} = 12$ and $M_T^{[1]} = M_T^{[2]} = M_T^{[3]} = M_R^{[2]} = M_R^{[3]} = 20$, the DoF that can be achieved by IA-XC algorithm is $\{D_{tr}\}$, where $D_{tr} = 2\ \forall r = 1$ and $D_{tr} = 4\ \forall r \neq 1$.

For the wireless channel with 3 transmitters and 3 receivers, there are three choices to achieve the transmission goal that each transmitter has distinct signals for each receiver: 1) we treat the wireless channel as a 3×3 MIMO XC and use IA-XC algorithm to select decoding and precoding matrices; 2) we treat the wireless channel as 3 3-receiver MIMO BCs and use IA-BC algorithm to select decoding and precoding matrices; 3) we treat the wireless channel as 3 3-user MIMO ICs and use IA-IC algorithm to select decoding and precoding matrices. To evaluate the performance in terms of transmission rate of our proposed IA-XC algorithm on asymmetric MIMO XCs, we compare the transmission method by using IA-XC algorithm with the other two transmission methods by using IA-BC and IA-IC algorithms, respectively. The simulation results are shown in Fig. 10. From the figure we can see that on both of these two asymmetric 3×3 wireless channels, when the transmission power is larger than 10dB, the IA-XC algorithm achieves higher sum rates than the other two algorithms.

VI. Conclusion

In this paper, we study IA problem of MIMO XC. We define the IA Feasible Conditions for the general MIMO XC, which includes IA Conditions and Rank Conditions. Due to the characteristics of the MIMO XC (i.e., each transmitter has distinct signals to each receiver), the desired signals for each receiver are from different transmitters, which makes the IA Conditions and the Rank Conditions coupled. As a result, the Rank Conditions of XCs cannot be automatically satisfied with probability one and removed directly as those of ICs. In order to deal with this problem, we first find the necessary conditions (i.e., Specific Rank Conditions) of IA Feasible Conditions and replace Rank Conditions with them, then prove that under Rewritten Feasible Conditions, Rank Conditions are satisfied almost surely. The Rewritten Feasible Conditions simplify the IA problem of general MIMO XC. Next, we design an iterative algorithm of IA schemes for general MIMO XCs. The IA-XC algorithm is efficient for general MIMO XC with limited signaling dimensions and has a good performance even under limited SNR.

Appendix

A. Proof of Lemma 1

We prove Eq. (8) first. Since $\boldsymbol{\mathcal{V}}_t \in \mathbb{C}^{M\times RD}$, we have rank$(\boldsymbol{\mathcal{V}}_t) \leq \min\{M,\ RD\}$, which implies that rank$(\boldsymbol{\mathcal{V}}_t) \leq RD$. If rank$(\boldsymbol{\mathcal{V}}_t) = RD$, then $M \geq RD$. We prove (8) by contradiction. Recall that $\mathbf{V}_{tr} \in \mathbb{C}^{M\times D}$ is with orthonormal columns, thus we have rank$(\mathbf{V}_{tr}) = D$ for all $t \in \mathcal{T}$ and $r \in \mathcal{R}$. If rank$(\boldsymbol{\mathcal{V}}_t) < RD$, there must be at least one precoding matrix $\mathbf{V}_{tr}$ such that:

$$\text{rank}([\boldsymbol{\mathcal{V}}_{t(-r)}, \mathbf{V}_{tr}]) < \text{rank}(\boldsymbol{\mathcal{V}}_{t(-r)}) + \text{rank}(\mathbf{V}_{tr}), \quad (28)$$

where $\boldsymbol{\mathcal{V}}_{t(-r)} \triangleq [\mathbf{V}_{t1}, \mathbf{V}_{t2}, \ldots, \mathbf{V}_{t(r-1)}, \mathbf{V}_{t(r+1)}, \ldots, \mathbf{V}_{tR}]$. As assumed in Sec. II-A, $\mathbf{H}_{tr} \in \mathbb{C}^{M\times M}$ is a full rank matrix. Hence, rank$(\mathbf{H}_{tr}\mathbf{W}) =$ rank$(\mathbf{W})$ for any matrix $\mathbf{W} \in \mathbb{C}^{M\times N}$ (N can be any positive integer) [33]. Together with (28), we have

$$\text{rank}([\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}, \mathbf{H}_{tr}\mathbf{V}_{tr}]) < \text{rank}(\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}) + \text{rank}(\mathbf{H}_{tr}\mathbf{V}_{tr}), \quad (29)$$

which indicates that some columns of $\mathbf{H}_{tr}\mathbf{V}_{tr}$ are linearly dependent on $\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}$. What's more, for fixed $\mathbf{U}_{tr}$, from IA Conditions (5) we have: $\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tn} = 0,\ \forall n \in \mathcal{R}, n \neq r$ (i.e., $\mathbf{U}_{tr}^H\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)} = 0$), which means that the subspace occupied by $\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}$ belongs to the null-space of $\mathbf{U}_{tr}$. Since some columns of $\mathbf{H}_{tr}\mathbf{V}_{tr}$ are linearly dependent on $\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}$, they are also in the null-space of $\mathbf{U}_{tr}$. Then the rank of $\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tr}$ is less than D, which contradicts (6). Therefore, the rank of $\boldsymbol{\mathcal{V}}_t$ is exactly RD. What's more, we have $M \geq RD$, since $\boldsymbol{\mathcal{V}}_t \in \mathbb{C}^{M\times RD}$ and rank$(\boldsymbol{\mathcal{V}}_t) = RD$.

The proof of Eq. (7) is similar to Eq. (8), hence we omit this proof.

B. Proof of Lemma 3

For a certain Rank Condition: rank$(\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tr}) = D$ (where t and r are fixed), we can divide the conditions in (5), (7) and (8) into two parts:

$$\begin{aligned}\text{Part A}: &\ \mathbf{U}_{mr}^H\mathbf{H}_{tr}\mathbf{V}_{tn} = 0,\quad \forall m \in \mathcal{T},\ \forall n \in \mathcal{R},\ n \neq r,\\ &(\text{i.e., } \boldsymbol{\mathcal{U}}_r^H\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)} = 0).\\ &\mathbf{U}_{mr}^H\mathbf{H}_{tr}\mathbf{V}_{tr} = 0,\quad \forall m \in \mathcal{T},\ m \neq t,\\ &(\text{i.e., } \boldsymbol{\mathcal{U}}_{r(-t)}^H\mathbf{H}_{tr}\mathbf{V}_{tr} = 0).\\ &\text{Specific Rank Conditions: Eqs. (7) (8).}\\ \text{Part B}: &\ \mathbf{U}_{m\tilde{r}}^H\mathbf{H}_{\tilde{t}\tilde{r}}\mathbf{V}_{\tilde{t}n} = 0,\quad \forall m \in \mathcal{T},\ n \in \mathcal{R},\ \tilde{t} \in \mathcal{T},\\ &\tilde{r} \in \mathcal{R},\quad |m-\tilde{t}| + |n-\tilde{r}| \neq 0,\\ &|\tilde{t}-t| + |\tilde{r}-r| \neq 0,\end{aligned}$$

First, we show that if we select the decoding and precoding matrices under the conditions in Part A, the selected decoding and precoding matrices $\mathbf{U}_{tr}$ $\mathbf{V}_{tr}$ satisfy the Rank Condition (i.e., rank$(\mathbf{U}_{tr}^H\mathbf{H}_{tr}\mathbf{V}_{tr}) = D$) almost surely.

Since the Specific Rank Conditions (7) and (8) are included in Part A, we know that $\mathbf{U}_{tr}$ is linearly independent of $\boldsymbol{\mathcal{U}}_{r(-t)}$ and $\mathbf{V}_{tr}$ is linearly independent of $\boldsymbol{\mathcal{V}}_{t(-r)}$ under the conditions in Part A. Hence $\mathbf{U}_{tr}$ and $\mathbf{H}_{tr}\mathbf{V}_{tr}$ can be rewritten as:

$$\mathbf{U}_{tr} = \tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}\mathbf{S}_u^{[1]} + \mathbf{N}_u\mathbf{S}_u^{[2]},\quad \mathbf{H}_{tr}\mathbf{V}_{tr} = \tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}\mathbf{S}_v^{[1]} + \mathbf{N}_v\mathbf{S}_v^{[2]}, \quad (30)$$

where $\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}$ and $\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}$ are orthonormal basis of $\boldsymbol{\mathcal{U}}_{r(-t)}$ and $\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}$, respectively.[3] $\mathbf{S}_u^{[2]}$ and $\mathbf{S}_v^{[2]}$ are both full rank $D\times D$ square matrices. $\mathbf{N}_u$ and $\mathbf{N}_v$ are $M\times D$ matrices with orthonormal columns and satisfy

$$[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u]^H[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u] = \mathbf{I}_{TD},$$
$$[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v]^H[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v] = \mathbf{I}_{RD}.$$

Substituting (30) into the conditions in Part A, we obtain

$$\begin{aligned}0 &= \boldsymbol{\mathcal{U}}_r^H\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)} \quad (31)\\ &= [\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}\mathbf{S}_u^{[3]}, \tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}\mathbf{S}_u^{[1]} + \mathbf{N}_u\mathbf{S}_u^{[2]}]^H\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}\mathbf{S}_v^{[3]},\end{aligned}$$

$$0 = \boldsymbol{\mathcal{U}}_{r(-t)}^H\mathbf{H}_{tr}\mathbf{V}_{tr} = {\mathbf{S}_u^{[3]}}^H\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}^H(\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}\mathbf{S}_v^{[1]} + \mathbf{N}_v\mathbf{S}_v^{[2]}), \quad (32)$$

$$[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u]^H[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u] = \mathbf{I}_{TD}, \quad (33a)$$

$$[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v]^H[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v] = \mathbf{I}_{RD}, \quad (33b)$$

In (31) and (32), $\boldsymbol{\mathcal{U}}_{r(-t)}$ is rewritten as $\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}\mathbf{S}_u^{[3]}$ and $\mathbf{H}_{tr}\boldsymbol{\mathcal{V}}_{t(-r)}$ is rewritten as $\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}\mathbf{S}_v^{[3]}$, where $\mathbf{S}_u^{[3]}$ and $\mathbf{S}_v^{[3]}$ are full rank $(T-1)D\times(T-1)D$ and $(R-1)D\times(R-1)D$ span matrices, respectively. Hence, it can be shown that the conditions in Part A are equivalent to the following conditions:

$$\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}^H\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)} = 0,\quad \mathbf{N}_u^H\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)} = 0, \quad (34)$$

$$\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}^H\mathbf{N}_v = 0. \quad (35)$$

$$[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u]^H[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u] = \mathbf{I}_{TD}, \quad (36a)$$

$$[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v]^H[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v] = \mathbf{I}_{RD}, \quad (36b)$$

[3]In this paper, when we define a matrix (e.g., $\tilde{\mathbf{A}}$) as an orthonormal basis of matrix $\mathbf{A} \in \mathbb{C}^{M\times N}$ ($M \geq N$), the matrix $\tilde{\mathbf{A}}$ has the following properties: i) there must exist a full rank matrix $\mathbf{B} \in \mathbb{C}^{N\times N}$ such that $\mathbf{A} = \tilde{\mathbf{A}}\mathbf{B}$, and ii) $\tilde{\mathbf{A}}^H\tilde{\mathbf{A}} = \mathbf{I}_N$.

With arbitrary set of decoding matrices, if we can find the precoding matrices that satisfy the conditions in (34) and (36), then we know that $[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u, \tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}]^4$ is a matrix with orthonormal columns. This is because $\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}$ is in the null space of $[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u]$, and $[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u]$ is a matrix with orthonormal columns.

We use matrix $\mathbf{N}$ to denote an orthonormal basis of the null space of the matrix $[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u, \tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}]$. Hence, $[\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}, \mathbf{N}_u, \tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}]$ is a unitary matrix. The conditions need to be satisfied by $\mathbf{N}_v$ are:

$$\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}^H \mathbf{N}_v = 0, \quad [\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v]^H [\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v] = \mathbf{I}_{RD}. \quad (37)$$

In order to satisfy the condition $\tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}^H \mathbf{N}_v = 0$, the matrix $\mathbf{N}_v$ should be in the space spanned by $[\mathbf{N}_u, \tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}]$. Moreover, since $[\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}, \mathbf{N}_v]$ is with orthonormal columns, the matrix $\mathbf{N}_v$ should be in the space spanned by $[\mathbf{N}_u, \mathbf{N}]$. In conclusion, $\mathbf{N}_v$ can be any orthonormal matrix lying in the space spanned by $[\mathbf{N}_u, \mathbf{N}]$. Hence, without loss of generality, the matrix $\mathbf{N}_v$ can be represented as:

$$\mathbf{N}_v = \text{orth}([\mathbf{N}_u, \mathbf{N}]\mathbf{S}_v^{[4]}) = [\mathbf{N}_u, \mathbf{N}]\mathbf{S}_v^{[4]}\mathbf{O}_v^{[1]}. \quad (38)$$

$\mathbf{S}_v^{[4]}$ is an $M-(T+R-2)D \times D$ span matrix that can be represented as $\mathbf{S}_v^{[4]} = \begin{bmatrix} \mathbf{S}_v^{[4.1]} \\ \mathbf{S}_v^{[4.2]} \end{bmatrix}$, where the entries of $\mathbf{S}_v^{[4.1]} \in \mathbb{C}^{D\times D}$ and $\mathbf{S}_v^{[4.2]} \in \mathbb{C}^{M-(T+R-1)D\times D}$ are randomly and independently generated from continuous distributions. $\mathbf{O}_v^{[1]}$ is a full rank square matrix used to orthonormalize the product matrix $[\mathbf{N}_u, \mathbf{N}]\mathbf{S}_v^{[4]}$. Therefore, we have

$$\begin{aligned} &\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr} \\ &= \left(\mathbf{S}_u^{[1]H} \tilde{\boldsymbol{\mathcal{U}}}_{r(-t)}^H + \mathbf{S}_u^{[2]H} \mathbf{N}_u^H\right)\left(\tilde{\boldsymbol{\mathcal{V}}}_{t(-r)}\mathbf{S}_v^{[1]} + \mathbf{N}_v \mathbf{S}_v^{[2]}\right) \\ &= \mathbf{S}_u^{[2]H} \mathbf{N}_u^H [\mathbf{N}_u, \mathbf{N}]\mathbf{S}_v^{[4]}\mathbf{O}_v^{[1]}\mathbf{S}_v^{[2]} \\ &= \mathbf{S}_u^{[2]H} [\mathbf{I}_D, \mathbf{0}_{D\times M-(T+R-1)D}]\mathbf{S}_v^{[4]}\mathbf{O}_v^{[1]}\mathbf{S}_v^{[2]} \\ &= \mathbf{S}_u^{[2]H} \mathbf{S}_v^{[4.1]}\mathbf{O}_v^{[1]}\mathbf{S}_v^{[2]}. \end{aligned}$$

As denoted in equations (30) and (38), the matrices $\mathbf{S}_u^{[2]}$, $\mathbf{S}_v^{[2]}$ and $\mathbf{O}_v^{[1]}$ are all full rank square matrices. Hence, the rank of the product matrix $\mathbf{S}_u^{[2]H}\mathbf{S}_v^{[4.1]}\mathbf{O}_v^{[1]}\mathbf{S}_v^{[2]}$ is equal to the rank of $\mathbf{S}_v^{[4.1]}$. Since the entries of $\mathbf{S}_v^{[4.1]}$ are randomly and independently generated from continuous distributions, $\mathbf{S}_v^{[4.1]}$ has full rank with probability 1. In other words, under the conditions in Part A, the rank condition (i.e., $\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D$) is satisfied almost surely. Mathematically, we have $\Pr\big(\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D \big| \text{conditions in Part A are satisfied}\big) \approx 1$. In the following analysis, we will use $\mathcal{R}$ to represent the statement "$\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D$", use $\mathcal{A}$ to represent the statement "*conditions in Part A are satisfied*", and use $\mathcal{B}$ to represent the statement "*conditions in Part B are satisfied*".

Next, we will show that

$$\Pr\big(\mathcal{R}\big|\mathcal{A},\mathcal{B}\big) = \frac{\Pr\big(\mathcal{A}\big)\Pr\big(\mathcal{R}\big|\mathcal{A}\big)\Pr\big(\mathcal{B}\big|\mathcal{A},\mathcal{R}\big)}{\Pr\big(\mathcal{A},\mathcal{B}\big)} \approx 1. \quad (39)$$

Considering the conditions in Part B. We can find that the channel matrices $\{\mathbf{H}_{\tilde{t}\tilde{r}} | \ \forall \tilde{t} \neq t \text{ or } \tilde{r} \neq r\}$ do not appear in the conditions in Part A and the rank condition (i.e., $\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D$). This implies that the decoding and precoding matrices that satisfy the conditions in Part A and rank condition (i.e., $\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D$) are independent of the channel matrices in Part B. Therefore, the conditions in Part B are independent of the conditions in Part A and the rank condition (i.e., $\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D$). In other words, we have

$$\Pr\big(\mathcal{B}\big|\mathcal{A},\mathcal{R}\big) = \Pr\big(\mathcal{B}\big), \quad \Pr\big(\mathcal{A},\mathcal{B}\big) = \Pr\big(\mathcal{A}\big)\Pr\big(\mathcal{B}\big). \quad (40)$$

Substituting (40) into (39), we have $\Pr\big(\mathcal{R}\big|\mathcal{A},\mathcal{B}\big) = \Pr\big(\mathcal{R}\big|\mathcal{A}\big)$. Since $\text{rank}(\mathbf{U}_{tr}^H \mathbf{H}_{tr} \mathbf{V}_{tr}) = D$ is satisfied almost surely under the conditions in Part A, we can conclude that under IA Conditions (5) and Specific Rank Conditions (7) (8), the Rank Conditions (6) are satisfied with probability 1.

C. Proof of Lemma 4

Since $\mathbf{U}_{tr}^H \mathbf{U}_{tr} = \mathbf{I}_D$, all the unit vectors that belong to the subspace spanned by $\mathbf{U}_{tr}$ can be expressed as $\hat{\mathbf{u}}_{tr} = \sum_{d\in\mathcal{D}} \alpha_d \mathbf{U}_{tr}^{\star d}$, and all the unit vectors that belong to the subspace spanned by $\mathbf{U}_{mr}$ $(m \neq t)$ can be expressed as $\hat{\mathbf{u}}_{mr} = \sum_{d\in\mathcal{D}} \beta_d \mathbf{U}_{mr}^{\star d}$, where $\sum_{d\in\mathcal{D}} \alpha_d^2 = 1$ and $\sum_{d\in\mathcal{D}} \beta_d^2 = 1$. Under the conditions in (15) in Lemma 4, we have

$$\begin{aligned} &\hat{\mathbf{u}}_{tr}^H \hat{\mathbf{u}}_{mr} \hat{\mathbf{u}}_{mr}^H \hat{\mathbf{u}}_{tr} \\ &= |\sum_{d_1\in\mathcal{D}} \alpha_{d_1} \mathbf{U}_{tr}^{\star d_1 H} \sum_{d_2\in\mathcal{D}} \beta_{d_2} \mathbf{U}_{mr}^{\star d_2}|^2 \\ &\overset{(b)}{\leq} \sum_{i\in\mathcal{D}} \alpha_i^2 \sum_{j\in\mathcal{D}} |\sum_{d_2\in\mathcal{D}} \beta_{d_2} \mathbf{U}_{tr}^{\star j H} \mathbf{U}_{mr}^{\star d_2}|^2 \\ &\overset{(c)}{=} \sum_{j\in\mathcal{D}} |\sum_{d_2\in\mathcal{D}} \beta_{d_2} \mathbf{U}_{tr}^{\star j H} \mathbf{U}_{mr}^{\star d_2}|^2 \\ &\overset{(d)}{\leq} \sum_{j\in\mathcal{D}} \sum_{k\in\mathcal{D}} \beta_k^2 \sum_{l\in\mathcal{D}} |\mathbf{U}_{tr}^{\star j H} \mathbf{U}_{mr}^{\star l}|^2 \\ &\overset{(e)}{=} \sum_{j\in\mathcal{D}} \sum_{l\in\mathcal{D}} |\mathbf{U}_{tr}^{\star j H} \mathbf{U}_{mr}^{\star l}|^2 = \text{Tr}(\mathbf{U}_{tr}^H \mathbf{U}_{mr} \mathbf{U}_{mr}^H \mathbf{U}_{tr}), \quad (41) \end{aligned}$$

where (b) and (d) are from the Cauchy-Schwarz inequality, (c) and (e) are from the fact $\sum_{d\in\mathcal{D}} \alpha_d^2 = 1$ and $\sum_{d\in\mathcal{D}} \beta_d^2 = 1$. (41) implies that the square of inner product of any two vectors belonging to the subspaces spanned by different decoding matrices $\mathbf{U}_{tr}$ and $\mathbf{U}_{mr}$ $(m \neq r)$ is less than $\text{Tr}(\mathbf{U}_{tr}^H \mathbf{U}_{mr} \mathbf{U}_{mr}^H \mathbf{U}_{tr})$.

We can show that if the rank of the matrix $\boldsymbol{\mathcal{U}}_r$ (i.e., $[\mathbf{U}_{1r}, \mathbf{U}_{2r}, \ldots, \mathbf{U}_{Tr}]$) is less than TD, there must be T vectors $\{\hat{\mathbf{u}}_{tr} | \ t \in \mathcal{T}\}$ that belong to the same $(T-1)$-dimensional subspace. According to the minimum property of Regular Simplex [34], if there exist T unit vectors that belong to the same $(T-1)$-dimensional subspace, then we have

$$\sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{T}, m>t} \hat{\mathbf{u}}_{tr}^H \hat{\mathbf{u}}_{mr} \hat{\mathbf{u}}_{mr}^H \hat{\mathbf{u}}_{tr} \geq \frac{T}{2(T-1)}. \quad (42)$$

Substituting (42) into (41), we have $\sum_{t\in\mathcal{T}} \sum_{m\in\mathcal{T}, m>t} \text{Tr}(\mathbf{U}_{tr}^H \mathbf{U}_{mr} \mathbf{U}_{mr}^H \mathbf{U}_{tr}) \geq \frac{T}{2(T-1)}$,

[4]We use $\text{orth}(\mathbf{A})$ to denote an orthonormal basis of the matrix $\mathbf{A}$.

which contradicts (15). Therefore, the conditions in (15) (see Lemma 4) are sufficient conditions of $\text{rank}(\boldsymbol{\mathcal{U}}_r) = TD\ \forall r$ (14b).

Similarly, we can obtain that the conditions in (16) are the sufficient conditions of (14c).

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